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Collective choice behavior: nonparametric characterization

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door

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When I was 18 years old, most people advised me to study economics. After a quick look at the program, I concluded that this was not an option, given that this also implied studying French. Today, I finished my PhD in economics and I am planning to teach in French. Clearly, this was not what I had expected twelve years ago. Of course this does not mean that I regret the choices that I have made. On the contrary, it only shows that along the way many people influenced my 'path of life'.

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erature on collective models, and Prof. Ian Crawford, currently one of the specialists in the revealed preference methodology, is a great honor. Their enthusiastic reactions have impressed me and are of course extremely motivating. I am very grateful for the time they have invested in this work and, as several times being indicated in this dissertation, I hope to present them in the near future some follow-up research based on their suggestions.
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Bram

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General introduction

According to Varian (1982), applied demand analysis typically addresses three sorts of issues concerning the behavioral model. [Varian (1984) suggests analogous issues in a production setting.] (i) Test consistency with the behavioral model under study. Given consistency with the behavioral model: (ii) recover consumer preferences (or the underlying technical constraints in the production setting); (iii) forecast behavior in other economic environments (e.g. other price configurations).

The consistency issue is of course the crucial one, since we first need a characterization of the behavioral model in order to be able to tackle the other two issues. The objective in this work is this first methodological step, in which we model *collective rationality*; as such we develop nonparametric tests for the collective model of multi-person household consumption behavior (see Chiappori, 1988 and 1992, and Varian, 1982, for seminal work). Given this approach, some may argue that we are 'non-standard' for two reasons. Firstly, we do not opt for the traditional *unitary* approach for modeling the household consumption behavior. This standard approach assumes that the household acts as if it were a single-decision maker that is maximizing his/her preferences; as such, this approach ignores the individual preferences of the household members and the intrahousehold decision process. Secondly, most behavioral models are *parametric* in nature, meaning that they rely on 'ad-hoc' (or 'non-verifiable') functional specifications of the preferences and the decision process. Below we argue that both standard approaches have serious deficiencies. We avoid these deficiencies by using a collective model and revealed preference axioms for testing the consumption behavior of multi-person households.

The *collective model* explicitly recognizes that a household is formed by individuals with own, possibly different, rational preferences. These individuals are assumed to engage in a bargaining process that results in a Pareto efficient intrahousehold allocation. So, instead of assuming a specific bargaining strategy, such as, e.g., the one underlying Nash bargaining or Kalai-Smorodinsky, the collective model starts from the sole assumption that the outcome should be Pareto efficient. Although this starting point incorporates the 'cooperative' strategies (since these typically result in Pareto efficient outcomes), it of course excludes some noncooperative strategies (which sometimes lead to inefficient outcomes). Nevertheless, since Pareto efficiency implies that there is 'no money left on the table', Chiappori argues that the Pareto efficiency assumption naturally extends the traditional individual rationality requirement towards *collective rationality*; by exhausting the budget the members try to maximize their preferences (see Chapter 1 for an extended discussion; see also Chiappori, 1988, and Chiappori and Ekeland, 2005).

In this work we mainly focus on the consumption behavior of households, but, straightforwardly, our results also hold for the decision process of *groups* of individuals. Of course, only for those groups where the Pareto efficiency assumption is justifiable or can at least be used as a benchmark case (see Chiappori and Ekeland, 2005 and 2006, for an extended discussion).

The fact that the collective approach takes individual preferences (and not 'household preferences') as the starting point, makes it, contrary to the unitary approach, particularly useful for addressing methodological (welfare-related) questions that specifically relate to the within-household distribution of the household means. A first step in addressing such questions (which is of course the ultimate goal of the theory) is the characterization and testing of the household behavior. However, to obtain concrete answers to such welfare-related questions, one also needs to identify the structural model underlying the observed behavior. In this work, we only focus on the first step, namely characterizing and testing the collective model; but, of course, this characterization opens the avenue for tackling recovery and forecasting questions in the (near) future.

In his original work, Chiappori (1988) focuses on a restricted collective model. He considers a labor supply setting that involves a number of convenient simplifications for the empirical analyst, such as observability of the individuals' labor supply (or leisure), egoistic agents and the exclusion of public consumption within the household.[1]

Browning and Chiappori (1998) provide a characterization of a general collective model that includes public consumption and preferences which allow for altruism and other externalities. Moreover, they start from the 'minimalistic' assumption that the empirical analyst cannot determine which goods are privately and/or publicly consumed within the household, and, that the quantities that are privately consumed by the different household members cannot be observed. As such they avoid the above mentioned simplifications and obtain a general collective model. Browning and Chiappori (1998) derive a necessity condition for collectively rational household behavior, while Chiappori and Ekeland (2006) address the associated sufficiency question. Contrary to the unitary model, the collective model entails empirical restrictions that seem more difficult to reject when tested on multi-person household data (see, e.g., Fortin and Lacroix, 1997, Browning and Chiappori, 1998, Vermeulen, 2005, and Cherchye and Vermeulen, 2007).

One inherent difficulty in the usual testing of behavioral models (as, e.g., the ones mentioned above) is that they implicitly rely on non-verifiable assumptions concerning the functional structure of preferences and, if needed, the intrahousehold bargaining process. Therefore, such standard *parametric* tests do not only check a model's theoretical implications, but also the ad-hoc functional specifications that are assumed.

By definition, *nonparametric* tests do not assume any functional specification regarding the household consumption process. They directly test the adequacy of a theory on the raw quantity and price data by means of revealed preference axioms. Building further on the seminal work of, among others, Afriat (1967), Varian (1982) conceived a necessary and sufficient condition for individual rationality, which is captured in the well-known *Generalized Axiom of Revealed Preference* (*GARP*). See, e.g., Varian (2006), for an up-to-date overview of the revealed preference methodology and Blundell, Browning and Crawford (2003, 2007) for recent developments.

Nonparametric tests of the collective model have been very scarce up to now. Along with parametric restrictions, Chiappori (1988) also derived nonparametric implications of his collective labor supply model

[1] To be precise, we have to note that Chiappori also considered a more general case by dropping the assumption of egoistic agents.

under study. Based on this work, Snyder (2000) and Cherchye and Vermeulen (2007) developed nonparametric tests for this restrictive collective model. Since this labor supply setting involves a number of convenient simplifications, we want to develop and apply nonparametric tests of general collective models that do not rely on these restrictive assumptions.

In the first chapter we introduce the two main features of this dissertation. We start by discussing the collective approach for modeling household consumption behavior by comparing it to alternative approaches. Secondly, we introduce *GARP* which forms the basis for nonparametric tests of individual rationality. As such, we present the necessary background for understanding the results in Chapters 2 to 4. We end Chapter 1 by showing how we can apply the insights of the collective approach to analyze the cost efficiency of multi-output firms.

The core chapter of this dissertation is Chapter 2, in which we aim at generalizing Chiappori's work (1988,1992) by providing a nonparametric characterization of the collective consumption model of Browning and Chiappori (1998), which includes both public consumption and externalities. Besides this nonparametric characterization, we also derive testable necessary and sufficient conditions for data consistency with this general model. As we will explain, these latter conditions are solely based on *observable* aggregate information. So, in this chapter we obtain our nonparametric tests for the general collective model of household consumption behavior.

In Chapter 3 we apply these nonparametric tests to data drawn from the Russia Longitudinal Monitoring Survey. The panel structure of this data set allows us to apply the tests to each separate household; as such we avoid the assumption of homogeneous preferences across households. Although our sample covers time series of only eight observations, there is, as we will show, enough relative price variation to meaningfully test behavioral models. In this way, we obtain a first nonparametric test of a general collective model, with public consumption and externalities, that uses 'real-life' data.

Next, since the general collective model cannot be rejected by the data at hand, we further propose in Chapter 3 a novel approach to model restricted versions of the general collective model. Specifically, we consider the possibility of including alternative positions regarding the *sharing rule* that applies to each household. This central concept in collective models captures the distribution of the household means over the household members. In fact, as we will demonstrate, this approach also obtains the unitary model as a special case of the general model. In our empirical application we nonparametrically test these plausible, but more restrictive, alternatives of the general collective consumption model, which imply alternative prior assumptions regarding the structure of the sharing rule. Besides presenting the test results, we also include a power analysis (like the one of Bronars, 1987) for the different specifications of the collective model.

In Chapter 4 we present a second application of our nonparametric tests of collective models, but now on the basis of experimental data. Our empirical application mainly focuses on an 'egoistic model', i.e. a collective model with egoistic individuals and no public consumption. Of course, when this restrictive collective model is rejected, we also consider the general model. In this chapter we want to investigate the 'appropriateness' of the egoistic model, since empirical applications of collective models mostly assume this restrictive model; the parsimonious nature of the model allows for a more powerful empirical analysis. For our laboratory test, we consider an unsophisticated consumption setting, involving a very limited number of commodities and a low budget. The underlying argument is that, if the egoistic model is to hold in the more sophisticated settings, then it must certainly hold for this unsophisticated setting.

The laboratory nature of experiments effectively avoids the usual preference heterogeneity and data problems. In fact, it has been argued that the nonparametric testing tools are especially useful within an experimental context; a particularly convincing case is provided by Sippel (1997), who focuses on individual rationality. Moreover, the experimental set-up allows us to obtain extra information on consumption quantities for the individual group members. Up till now, 'real-life' data sets mostly only contain aggregate household quantities and therefore do not reveal the individual members' consumption quantities. As we will show, this additional information allows for more powerful tests of the collective consumption model.

Our study complements the existing nonparametric-experimental literature that focuses on the appropriateness of the utility maximization model for describing rational individual behavior; see, e.g., Sippel (1997), Harbaugh, Krause and Berry (2001), Andreoni and Miller (2002) and references therein.

In the first chapters we consider a bargaining context in which individuals maximize their utility given the household budget. Browning and Chiappori (1998) present a dual representation of this maximization program by adapting the dual reformulations for individuals towards households. More precisely, duality implies that rational individuals minimize their expenditures given their utility level; analogously collectively rational households minimize their expenditure given both the 'household utility' and the bargaining power (see, e.g., Browning and Chiappori, 1998, for more details).
In the final Chapter 5, this reformulation of collective rationality forms the basis for applying the collective approach to the cost efficiency of multi-output firms. Indeed, firms are rational if they are expenditure minimizing (or cost efficient). We exploit the insights of Chapter 2 to provide a nonparametric characterization of cost efficient behavior of multi-output production firms. We take as starting point that in the cost minimization process such firms benefit from economies of scope, which in turn originate from joint input use and input externalities. Economies of scope can be loosely defined as situations where the average total cost of production decreases as a result of increasing the number of different goods produced (see, e.g., Baumol, Panzar and Willig, 1982). Our approach extends the (nonparametric) results for single-output firms (see Varian, 1984) by exploiting the scope economies in the multi-output production process.
We stress that we do not present a methodology for investigating the extent to which economies of scope are actually present. Rather, we present a method for analyzing cost efficient production behavior that exploits a number of specific features related to scope economies. As we will show, these tests are directly linked to the nonparametric efficiency assessment literature known as Data Envelopment Analysis (see, e.g., Charnes, Cooper and Rhodes, 1978, and Cooper, Seiford and Tone, 2000, for an introduction).
We end this chapter by applying our methodology to assess the cost efficiency of research programs in Economics and Business Management Faculties of Dutch universities. This application shows that our method results in a more powerful efficiency analysis, as compared to standard methods, such as, e.g., the method of Cherchye and Vanden Abeele (2005).

Finally, note that the different chapters of this thesis were first written as separate papers (except for the preliminary Chapter 1). While we have tried to keep the repetition of the arguments to a strict minimum, we also included some ‘functional’ overlap, in order to improve the exposition of the ideas. Moreover, by repeating the crucial concepts in alternative ways, we hope to better sketch the intuition behind them.

Chapter 1

The collective approach and revealed preferences: setting the stage

Abstract

In this chapter we introduce the several concepts that we will use in this dissertation. We start by discussing the collective approach and present the general collective model of Browning and Chiappori (1998). Secondly, we introduce the Generalized Axiom of Revealed Preferences ($GARP$), which forms the basis for nonparametric demand analysis. Finally, we explain the conceptual analogy between consumption of multi-person households and multi-output production. As such we provide the necessary background for understanding the next chapters.

1.1 Introduction

The goal of this dissertation is to develop and apply nonparametric tests of collective choice behavior for both consumption and production settings. In this chapter we introduce and motivate the several concepts that are related to this objective; as such we hope to provide the necessary intuition to understand the results of the next chapters.

The main part of this thesis consists of analyzing household consumption behavior for multi-person households given their restricted budget. In the literature, we can broadly identify three different approaches to model this behavior (see, e.g., Vermeulen, 2002, Donni, 2007, and Lundberg and Pollak, 2007, for general overviews).
Firstly, there is the standard *unitary approach* which assumes that the (multi-person) household acts as if it were a single decision maker; it maximizes a well-behaved (single) utility function subject to a household budget constraint. This approach therefore implies that the individual preferences of the members can be aggregated into a single household utility function, which is of course a strong assumption.

A totally different approach, which acknowledges the individual preferences of the household members, are the models that start from *strategic behavior*; each individual tries to maximize his or her own utility function given the strategy of the other individuals. Although this approach explicitly recognizes the individual preferences, it is again not completely satisfactory given the context of household behavior for which we may expect that there is (some) cooperative behavior. Indeed, strategic approaches may lead to Pareto inefficient outcomes, implying that it is possible to make one member better off without making the other members worse off.

The *collective approach* simultaneously deals with the above remarks by explicitly recognizing that the individual household members have own, possibly different, rational preferences and that these individuals enter into a bargaining process to divide the available budget. The sole assumption on this bargaining process is that it results in a Pareto efficient outcome. In Section 1.2 we discuss more formally the three different approaches and, as such, we motivate our choice for the collective approach.

Most empirical studies of behavioral models, such as, e.g., the ones introduced in Section 1.2, are typically *parametric* in nature; i.e. they critically rely on a (non-verifiable) functional structure for representing the individual preferences and, if needed, the bargaining process. This implies that standard parametric tests do not only check a model's theoretical implications, but also the ad-hoc functional specifications that are assumed. A rejection of a behavioral model may thus well be due to misspecifications. In this dissertation, we follow a different approach, labeled the *nonparametric* approach, which analyzes household behavior without imposing any parametric structure on e.g. preferences. Instead, using *revealed preference* axioms, the nonparametric approach tests the restrictions of the behavioral model directly on the 'raw' price-quantity data. See, e.g., Afriat (1967), Varian (1982) and, more recently, Blundell, Browning and Crawford (2003, 2007). As such, our tests are more robust since they cannot be influenced by misspecifications of the functional forms. In Section 1.3 we present this nonparametric methodology.

As already mentioned, we do not only consider consumption settings, but we also apply the collective approach to a multi-output production setting. In Section 1.4 we argue that the results of the collective model can indeed be easily adapted towards this production setting. Moreover, we show that the extra information, contained in the firms observed outputs (as counterpart to the unobserved individual utilities in the consumption context), enables us to develop powerful tests for analyzing the efficiency of these multi-output firms.

1.2 Modeling consumption behavior of multi-person households

In the following sections we consider alternative ways to characterize the consumption behavior of multi-person households. Suppose we observe T choices of n-valued bundles. For each observation j the vector $\mathbf{q}_j \in \mathbb{R}^n_+$ denotes the chosen quantities under the prices $\mathbf{p}_j \in \mathbb{R}^n_{++}$; and $S = \{(\mathbf{p}_j; \mathbf{q}_j), j = 1, ..., T\}$ represents the set of all observations for the household under study.

1.2.1 The unitary approach

The standard *unitary model* assumes that the household acts as a single decision maker who tries to maximize its preferences. A necessary and sufficient condition for the observed behavior to be consistent with the utility maximization hypothesis is that there exists a (nonsatiated) utility function U that *rationalizes* the data, i.e. for all $j = 1, \ldots T$ the value $U(\mathbf{q}_j)$ equals

$$\max_{\mathbf{q} \in \mathbb{R}^n_+} U(\mathbf{q}) \text{ s.t. } \mathbf{p}_j'\mathbf{q} \leq \mathbf{p}_j'\mathbf{q}_j. \tag{1.1}$$

This maximization problem leads to demand functions which have the well-known properties: (i) adding up, (ii) homogeneity, (iii) symmetry and negative semi-definiteness of the Slutsky matrix. Although these properties are very convenient in practice, they are, especially the symmetry condition, often rejected in *empirical studies*. See, e.g., Deaton and Muellbauer (1980), Blundell (1988), Browning and Meghir (1991), Fortin and Lacroix (1997), Browning and Chiappori (1998), Vermeulen (2005) and Cherchye and Vermeulen (2007).

While the above empirical evidence focuses on rejection of the properties of the demand functions, there is another implication of the unitary model that is clearly rejected by the data. The straitjacket of the unitary model also implies the *income pooling hypothesis*, which means that the source of the household income should not have any effect on the intrahousehold allocation. Not surprisingly, this restriction is also clearly rejected by the data. See, e.g., Thomas (1990), Bourguignon et al. (1993), Browning et al. (1994), Lundberg et al. (1997) and Fortin and Lacroix (1997).

Besides this overwhelming empirical evidence, the unitary approach is mainly criticized on methodological grounds. By assuming that the household acts as a single agent, or equivalently by assuming 'household preferences', the unitary model ignores the principle of *methodological individualism* which states that social theories should run in terms of individuals, rather than in terms of groups of individuals (see, e.g., Blaug, 1980). As such the unitary model does not take into account that the individuals in the group have different preferences, and that among them an intrahousehold decision process takes place. Of course, the distinction between household and individual preferences becomes irrelevant if all members have the same preferences (and therefore there is no decision process needed) or if the household preferences coincide with the individual preferences of one member (which then acts as a 'paternalistic dictator'). In all other scenarios, the household preferences can be seen as the aggregate of the individual preferences. This is a strong assumption, given that since the well-known impossibility theorem of Arrow (see, e.g., Arrow, 1963), we know that we need extra (restrictive) conditions to aggregate individual preferences (see, e.g., Pollak, 2007 for discussion) . A notable example are the Gorman aggregation conditions, from which one can derive that, because of the quasi-homothetic preferences, the Engel curves are linear in terms of the total expenditures (see, e.g., Deaton and Muellbauer, 1980 for an extensive discussion). Clearly this last implication is not what we observe in the data (see, e.g., Vermeulen, 2002, Donni, 2007, and Lundberg and Pollak, 2007, for discussion).

As we will show in Section 1.2.3 the collective approach conveniently deals with the above drawbacks of the unitary model. Starting from individual preferences, it will derive testable properties of demand functions and avoid the income pooling hypothesis. In general, any model that acknowledges the individual preferences of the household members (such as, e.g., the collective model), allows for addressing (normative) *welfare related questions* concerning the intrahousehold allocation.

A first type of such questions fits within the 'targeting view' of Blundell, Chiappori and Meghir (2005), which states that the effectiveness of a specific benefit or tax also depends on the particular household member to whom it has been targeted. These authors argue that the collective model is well equipped to investigate such statements, while the unitary model, since it implies income pooling, can of course not deal with such considerations.

Related to this is the recovery of the so-called 'sharing rule' that divides the aggregate household means over the individual household members (see Browning, Chiappori and Lewbel, 2006, for a discussion of this sharing rule concept in a parametric treatment of the collective consumption model). Recovering this sharing rule, and subsequently explaining its variation in terms of household (member) characteristics, can yield useful insights into the distribution of the within-household bargaining power across the individual household members (see, e.g., Browning et al., 1994; Browning and Chiappori, 1998; Chiappori and Ekeland, 2006). This issue is irrelevant in the unitary approach, given that it does not model the intrahousehold decision process.

Another type of questions follows from the fact that models starting from individual preferences allow for analyzing welfare at the individual household member level rather than at the aggregate household level. For instance, Browning, Chiappori and Lewbel (2006) suggest a collective approach for comparing the cost of living of individuals living alone with that of the same individuals living in a multi-member household. Again, the unitary model can not tackle such questions, since it ignores the individual preferences.

1.2.2 The strategic approach

The strategic approach deals with the criticism that the unitary model does not start from individual preferences; it assumes that household members maximize their individual utility, given the optimal strategy of the other members. See, e.g., Ulph (1988), Browning (2000), Chen and Woolley (2001), Lechene and Preston (2005, 2007) for applications. Of course, this Nash equilibrium setting implies other restrictions on the household behavior than the unitary approach. Moreover these restrictions often result in rather weak tests of the behavioral model, certainly in comparison to the unitary model.

The main implication is of course that noncooperative models may result in Pareto inefficient intrahousehold allocations, implying that in such cases, individuals can be made better off without making other members worse off. Although there are many settings that indeed lead to Pareto inefficient outcomes, there are some intuitive arguments which support that household consumption behavior is not an example of such a setting. Firstly, assuming Pareto efficient outcomes imply that no member can be made better off without making another member worse off. So the members use all opportunities for Pareto improvements, which is also referred to as there is 'no money left on the table'. As such, Pareto efficiency seems the natural generalization of individual rationality towards our multi-person settings. Indeed, assuming Pareto efficiency means that the household is *collectively rational* since the household members exhaust the budget to maximize their utility (see, e.g., Chiappori, 1988).

In order to present more 'game theoretic' arguments, we start by noting that household consumption behavior is a nice example of a repeated game. Indeed, the households have to take, on a regular basis, the same consumption decisions (e.g. the expenditures on nondurables). This repetitive character justifies, at least to some extent, to assume that all members in the household know the preferences of the other members or, more generally, that there is symmetric information. In such a setting, bargaining (and cooperative) models typically lead to Pareto efficient outcomes. In our opinion, these *cooperative* techniques form the natural starting point for modeling multi-person household

consumption behavior. At least if we assume that the members voluntarily form a household and can therefore use the threat to leave the household.
Secondly, also in the *noncooperative* framework we may expect Pareto efficient outcomes. It is well known that in repeated games with symmetric information, cooperation often emerges as the long run equilibrium. Certainly if one considers an infinite time horizon, which is again tenable in our setting.
A final game theoretic argument is the fact that, in our opinion, it seems too restrictive to assume that there is no communication in the 'noncooperative games' used to model household behavior. Note that communication also includes the possibility to sign contracts in order to force commitment or to make threats (such as e.g. leaving the household) credible. Again, games with communication make it easier to support Pareto efficient outcomes. See, e.g., Aumann and Hart (1994) and Myerson (1997) for more formal discussions concerning these 'game theoretic' arguments.

We believe that the above, only intuitive, arguments show that it is plausible to assume that the members in the household find mechanisms to support Pareto efficient outcomes. If not, we can at least use the collective approach to verify this statement: if the data rejects the restrictions of the collective approach, one may interpret this as rejection of the Pareto efficiency assumption. In this respect, we also refer to the formal discussion of Pareto efficiency in Lechene and Preston (2007). Using a 'household Nash equilibrium model', they show that their strategic model leads to different restrictions on the household demand than the collective models (see also our discussion at the end of Section 1.2.3). We can therefore use the collective model as a benchmark, namely what the members could have obtained if they reached a Pareto efficient outcome.
Finally, to end this plea, most studies seem to suggest that the collective model better fits the data. See, e.g., Fortin and Lacroix (1997), Browning and Chiappori (1998) Vermeulen (2005), Cherchye and Vermeulen (2007) and our applications in Chapter 3 and 4. One exception is the study of Udry (1996) on African agricultural data; see also Akresh (2005), for a recent discussion, which challenges Udry's conclusions. Nevertheless, we believe that it is certainly interesting to investigate the restrictions one may derive from assuming Pareto efficient behavior.

1.2.3 The collective approach

As argued in the previous sections, in order to model multi-person household consumption behavior we need a model which acknowledges the individual preferences of the members and leads to Pareto efficient outcomes. An obvious starting point for such models is the axiomatic bargaining theory; using specific bargaining rules, one specifies how he household members divide the gains of living together. See for instance Manser and Brown (1980), McElroy and Horney (1981) and McElroy (1990) for results concerning the Kalai-Smorodinsky and Nash bargaining solution.

However, an important drawback of choosing an explicit bargaining model is that, if the empirical restrictions are rejected, then it could be difficult to determine which specific bargaining rule (such as for instance Pareto efficiency) causes this rejection. Therefore, Chiappori (1988, 1992) introduces the *collective approach* in which he takes a common property of the bargaining strategies as starting point, i.e. the intrahousehold decision process results in a Pareto efficient outcome. In contrast to bargaining models, which result in a specific point on the Pareto frontier, Chiappori does not specify in which point on the Pareto frontier the household will end up. Now, given that most bargaining models assume Pareto efficient outcomes, all these models certainly have to satisfy the empirical restrictions of the *collective model*. Or equivalently, rejection of the collective model raises serious doubts about the bargaining (or cooperative) approach and henceforth the Pareto efficiency assumption.

The collective model introduced in Chiappori (1988,1992) restricts attention to a labor supply setting, which involves a number of convenient simplifications for the empirical analyst (e.g., observability of household members' leisure/labour supply and no public consumption). In this dissertation we focus on the more general collective model introduced by Browning and Chiappori (1998), implying that we include public consumption and preferences that allow for externalities (including altruism). Moreover, Browning and Chiappori also start from the (minimal) assumption that the empirical analyst only observes aggregate demands and prices (i.e. (s)he only observes the data set $S = \{(\mathbf{p}_j; \mathbf{q}_j), j = 1, ..., T\}$).

To introduce the model of Browning and Chiappori (1998), we focus on two-member households. The generalization towards M-person households is straightforward. Since the general collective consumption model allows for both externalities and public consumption inside the household, it is of course not the aggregate consumption bundle $\mathbf{q}$ that generates utility for the individuals, but the *disaggregated* intrahousehold allocation of this bundle:

$$\mathbf{q} = \mathbf{q}^1 + \mathbf{q}^2 + \mathbf{q}^h,$$

with $\mathbf{q}^1, \mathbf{q}^2 \in \mathbb{R}^n_+$ the (unobserved) private consumption quantities of members 1 and 2, and $\mathbf{q}^h \in \mathbb{R}^n_+$ the (unobserved) public consumption quantities. In principle, a consumption commodity can be used for private consumption as well as public consumption (or combinations of both). For example, a car can be partly used by member 1 to drive to his/her work (i.e. private consumption), and also partly used by the household to go on a family trip (i.e. public consumption).
Each individual has preferences, defined over these intrahousehold allocations, which are represented by the utility functions $U^m(\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^h)$ ($m = 1, 2$). Thus, both members' utilities depend on $\mathbf{q}^1$ and $\mathbf{q}^2$, which captures the possibility of consumption externalities, as well as on $\mathbf{q}^h$, which captures public consumption.

Using this, we follow Chiappori (1988) to define a necessary and sufficient condition for *collective rationality* (see also Browning and Chiappori, 1998, for a parametric counterpart). A pair of (monotonically increasing) utility functions U^1 and U^2 *collectively rationalize* the set S if and only if there exists for each observation $j = 1, \ldots T$: (i) disaggregated quantities $(\mathbf{q}^1_j, \mathbf{q}^2_j, \mathbf{q}^h_j)$ and (ii) a weight $\mu_j \in \mathbb{R}_{++}$, such that the value $U^1(\mathbf{q}^1_j, \mathbf{q}^2_j, \mathbf{q}^h_j) + \mu_j U^2(\mathbf{q}^1_j, \mathbf{q}^2_j, \mathbf{q}^h_j)$ equals

$$\begin{aligned} \max_{\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^h \in \mathbb{R}^n_+} \; & U^1(\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^h) + \mu_j U^2(\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^h) \\ & \text{s.t. } \mathbf{p}'_j(\mathbf{q}^1 + \mathbf{q}^2 + \mathbf{q}^h) \leq \mathbf{p}'_j \mathbf{q}_j. \end{aligned} \tag{1.2}$$

In words, the collective consumption model (1.2) generalizes the standard (individual) utility maximization model (1.1) by describing household behavior as maximizing a weighted sum of the individual member utilities. The weighting of the utilities exactly reflects the Pareto efficiency characterization of optimal intrahousehold allocations.
Following Browning and Chiappori (1998), we interpret the Pareto weights μ_j as representing the relative 'bargaining power' (*vis-à-vis*

member 1) of member 2. Since μ_j is observation dependent, this bargaining power may vary according to the specific observation/situation j at hand. A greater bargaining power implies, ceteris paribus, a higher utility level for the corresponding individual. This higher utility level is not necessarily due to a more favorable own private consumption bundle; it may also follow from the other members private consumption (through externalities) or from publicly consumed quantities.

In general, these bargaining weights may depend on the observed prices, total expenditures and the so-called distribution factors This implies that in the collective setting the individual utility of a member also depends on the prices.[1] This price dependence is crucial, since Browning, Chiappori and Lechene (2006) argue that it is necessary to distinguish the unitary from the collective model. As we will argue below, it forms the basis of the test of Browning and Chiappori (1998).
Distribution factors are variables that influence the bargaining weight, but do not directly influence the preferences. Examples are sex ratio, age difference between partners and income shares (see, e.g., Bourguignon, Browning and Chiappori, 2006, for a recent discussion). Importantly, Bourguignon et al. (1993) show that for two member households, all distribution should be collinear (or proportional), which is again a testable implication (see Browning and Chiappori, 1998, for an application).

Starting from this definition of collective rationality, Browning and Chiappori (1998) also derive restrictions on the demand functions and they introduce the *pseudo-Slutsky matrix*. Their core result for two-person households is that, under collective rational household behavior, the pseudo-Slutsky matrix can be written as the sum of a symmetric negative semi-definite matrix P and a rank one matrix R. The first part P is the traditional Slutsky matrix, which captures the changes in demand induced by the variation in prices (holding U^1, U^2 and μ constant); while the second part R reflects the changes in demand induced by the variation in the bargaining weights (holding U^1, U^2 constant). So, if there are no changes in the bargaining power, then R is not needed and they obtain the standard unitary results. Indeed, if there are no changes in the intrahousehold decision process, then we can aggregate (using a constant weight $\mu = \mu_j$) the preferences of the members in a (single) household utility function. On the other hand, if there are

[1] The collective model need not be the only way to model price dependent preferences. See, e.g., Pollak, 1977 for an alternative approach.

changes in the bargaining weight, there will be a deviation of the traditional Slutsky matrix (and so the unitary model will be rejected). This deviation can be captured by a rank one matrix, since the changes, induced by the variation in the bargaining weights, have to take place along the one-dimensional Pareto frontier. Browning and Chiappori (1998) show the necessity of this condition; Chiappori and Ekeland (2006) address the associated sufficiency question.

Clearly, this pseudo-Slutsky matrix allows for adapting the existing procedures for the unitary model towards the collective setting. More precisely, the result of Browning and Chiappori gives a possibility to test the adequacy of the collective model (and as such of the Pareto efficiency assumption). Indeed, we merely have to verify the rank of R: if it has rank zero, we can, based on this test, not reject the unitary model and the collective model; if it is has rank one, we reject the unitary model but not the collective model for two members; if it has a rank higher than one, then we reject both the unitary and the collective model for two members.[2] So, Browning and Chiappori (1998) indeed obtained the test of the Pareto efficiency assumption that we were looking for. Moreover, the results of Lechene and Preston (2007) for their 'household Nash equilibrium model' use this same pseudo-Slutsky matrix. They show that if the outcome of the decision process is not Pareto efficient, then the rank of R is higher than one (depending on the number of public goods). We refer to Lechene and Preston (2007) for more details.

To end this section, we want to stress that all models discussed in this work are *static* in nature, implying that we ignore *dynamic* effects such as for example habits formation and intertemporal decision making. See, e.g., Browning, 1989, and Crawford, 2007 for nonparametric results in an unitary setting, Mazzocco, 2007 for parametric results for the collective model and Spinnewyn, 1981, for a formal link between habits formation and intertemporal decision making. Combining these insights with our results will be an important line of future research.

1.3 Nonparametric methodology

The tests introduced in the previous section are all examples of parametric tests. Indeed, to perform them we have to specify demand func-

[2] Note that this implies that we, like in Browning, Chiappori and Lechene (2006), claim that the unitary model is rejected as soon as Slutsky symmetry is rejected.

tions (reflecting the specific preferences), estimate the needed parameters by use of the data and verify the properties of the (pseudo-)Slutsky matrix. Of course, these results are satisfactory only if the parametric form, of for instance the preferences, is indeed the one that underlies the observed data. For instance, if one rejects the symmetry of the Slutsky matrix, this could be due to rejection of the unitary approach, but also due to a misspecification of the parametric form. In order to obtain more robust results, we want to avoid such specifications of functional forms.

By definition, *nonparametric tests* require no 'ad-hoc' functional specifications of demand functions. Using the concept of *revealed preferences* they test the adequacy of the behavioral model, i.e. preference maximization, on the raw price-quantity data. See, e.g., Varian (2006) for an up-to-date overview of the revealed preference methodology and Blundell, Browning and Crawford (2003, 2007) for recent developments. In this section we introduce the nonparametric characterization for individual rationality; in Chapter 2 we establish analogous results for the general collective model of Browning and Chiappori (1998). Note that, since the unitary approach assumes that the household acts as a single agent, it results in the same nonparametric tests as the ones for individual rationality.

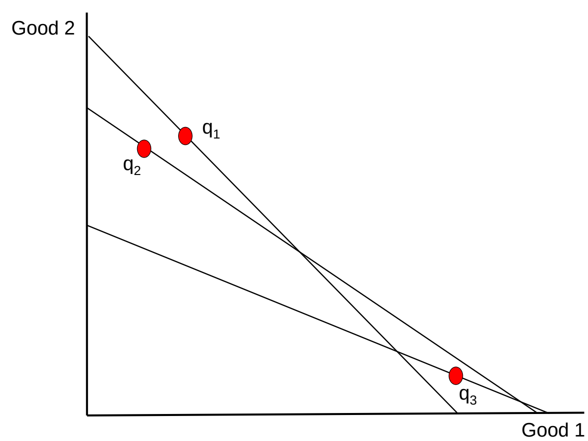
Samuelson (1938, 1948) provided the first nonparametric tests for individual rationality by describing, what is now known as, the Weak Axiom of Revealed Preferences (WARP) for situations with 2 goods. Houthakker (1950) generalized these tests toward settings with n goods by introducing, what is now called, the Strong Axiom of Revealed Preferences (SARP). Finally, based on the seminal work of Afriat (1967), Varian (1982) demonstrated that a data rationalizing utility function as in (1.1) exists if and only if the observed set S is consistent with the *Generalized Axiom of Revealed Preference* (*GARP*). For the sake of clarity, we will only focus on *GARP* and not on the alternative axioms such as WARP, SARP and strong SARP (see, e.g., Varian, 2006, and Chiappori and Rochet, 1987, for a formal discussion of the several concepts).

As we will show, *GARP* exploits the idea that a bundle $\mathbf{q}_j$ is utility maximizing subject to its budget constraint if and only if it is expenditure minimizing over its 'better than' set. While the expenditures are directly observed (i.e. $\mathbf{p}_j'\mathbf{q}_j$), the same does not hold for the 'better than' set. Therefore Varian approximates this latter set by using *revealed preference* relations.

Definition 1.1. *For a set of observations* $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, ..., T\}$*: if* $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$ *then* $\mathbf{q}_i R_0 \mathbf{q}_j$*; and if* $\mathbf{q}_i R_0 \mathbf{q}_k$*,* $\mathbf{q}_k R_0 \mathbf{q}_l, \ldots,$ $\mathbf{q}_z R_0 \mathbf{q}_j$ *for some (possibly empty) sequence* $(k, l, \ldots, z)$ *then* $\mathbf{q}_i R \mathbf{q}_j$*.*

The relation R_0 is commonly referred to as the *direct revealed preference* relation, while its transitive closure R is known as the *revealed preference* relation. In words, this definition states that we can infer from the data that a bundle $\mathbf{q}_i$ is directly revealed preferred over a bundle $\mathbf{q}_j$ (i.e. $\mathbf{q}_i R_0 \mathbf{q}_j$), if under the prices of observation i (i.e. $\mathbf{p}_i$) the latter bundle was attainable (i.e. $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$) but was not chosen. Subsequently, the revealed preference concept exploits that preferences are transitive: if one prefers $\mathbf{q}_i$ over $\mathbf{q}_k$ (i.e. $\mathbf{q}_i R_0 \mathbf{q}_k$) and $\mathbf{q}_k$ over $\mathbf{q}_j$ (i.e. $\mathbf{q}_k R_0 \mathbf{q}_j$), then transitivity implies that one also prefers $\mathbf{q}_i$ over $\mathbf{q}_j$ (i.e. $\mathbf{q}_i R \mathbf{q}_j$).
Example 1.2 illustrates these relations for a situation with two goods.

Fig. 1.1. Revealed preferences



Example 1.2. *Figure 1.1 shows three observations of bundles of two commodities. For each observation* i *the straight line (or budget line) through the bundle* $\mathbf{q}_i$ *represents the budget spent in observation* i*. This*

implies that all bundles on or below this line were feasible in observation i *(with respect to the budget* $\mathbf{p}_i'\mathbf{q}_i$ *and the prices* $\mathbf{p}_i$ *of the commodities in the given observation* i*).*
Given that bundle $\mathbf{q}_2$ *lies below the budget line through* $\mathbf{q}_1$*, this implies that under the prices* $\mathbf{p}_1$*,* $\mathbf{q}_1$ *is more expensive than* $\mathbf{q}_2$ *(i.e.* $\mathbf{p}_1'\mathbf{q}_1 \geq \mathbf{p}_1'\mathbf{q}_2$*). So we observe that in observation 1 bundle* $\mathbf{q}_2$ *was attainable (and cheaper), but nevertheless the individual chooses* $\mathbf{q}_1$*. Therefore we conclude that (s)he prefers* $\mathbf{q}_1$ *over* $\mathbf{q}_2$ *or* $\mathbf{q}_1 R_0 \mathbf{q}_2$*. Analogously we find that* $\mathbf{q}_2 R_0 \mathbf{q}_3$*.*
Since $\mathbf{q}_1$ *is preferred over* $\mathbf{q}_2$ *and* $\mathbf{q}_2$ *over* $\mathbf{q}_3$*, transitivity then implies that* $\mathbf{q}_1$ *should also be preferred over* $\mathbf{q}_3$ *or* $\mathbf{q}_1 R \mathbf{q}_3$*. Note that* $\mathbf{q}_3$ *does not belong to the budget set of observation 1, i.e. we do not have* $\mathbf{q}_1 R_0 \mathbf{q}_3$

Using Definition 1.1, we can now define the *Generalized Axiom of Revealed Preference* (*GARP*).

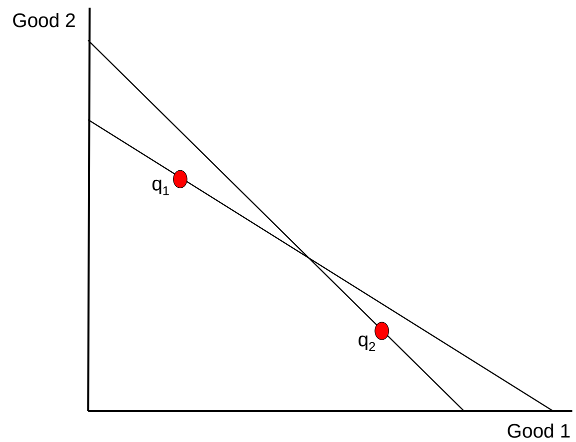
Definition 1.3. *A set of observations* $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, ..., T\}$ *satisfies* GARP *if* $\mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'\mathbf{q}_i$ *whenever* $\mathbf{q}_i R \mathbf{q}_j$*.*

In words this definition states that S satisfies *GARP* if and only if each observation j is expenditure minimizing over all the revealed preferred bundles $\mathbf{q}_i$. As such, the *GARP* indeed provides the basis for a test of *individual rationality* by expressing the idea that the bundle $\mathbf{q}_j$ is utility maximizing subject to its budget constraint if and only if it is expenditure minimizing over its 'better than' set. The corresponding test proceeds in two steps: one first recovers the relations R_0 and R, which define the (*in casu* revealed) 'better than' set for each $\mathbf{q}_j$ (i.e. the bundles $\mathbf{q}_i$ such that $\mathbf{q}_i R \mathbf{q}_j$); subsequently, one checks if $\mathbf{q}_j$ is effectively cost minimizing over that set, which requires $\mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'\mathbf{q}_i$ for $\mathbf{q}_i R \mathbf{q}_j$. Finally, note that there exist efficient algorithms for applying this test; a notable example is Warshall's algorithm based on Boolean matrices (see Warshall, 1962, and Varian, 1982).
Example 1.4 illustrates *GARP* (again for a situation with two goods).

Example 1.4. *Figure 1.2 shows a rejection of* GARP*. Indeed, from the figure we conclude that* $\mathbf{q}_1$ *is preferred over* $\mathbf{q}_2$ *(i.e.* $\mathbf{q}_1 R_0 \mathbf{q}_2$*) and we also observe that* $\mathbf{q}_2$ *is not cost minimizing since* $\mathbf{p}_2\mathbf{q}_2 > \mathbf{p}_2\mathbf{q}_1$*.*

Note that this example shows that it suffices to have two observations and two commodities to reject *GARP* (and thus also to reject individual rationality). We can easily show that this is also a necessary condition:

Fig. 1.2. Rejection of *GARP*



(i) if there is only one observation we cannot specify a 'better than' set, so a fortiori we cannot reject $GARP$; (ii) if there is only one commodity, then all multiplications in Definition 1.3 are scalar multiplications. This implies that $q_i R_0 q_j$ if and only if $q_i \geq q_j$ and thus also that $q_i R q_j$ if and only if $q_i \geq q_j$. Under this condition, a quantity bundle is of course always cost minimizing over its 'better than' set (and so, again, we cannot reject $GARP$.
So for one member we need 2 $(= 1+1)$ observations and 2 commodities to nonparametrically reject individual rationality. In Chapter 2 we extend this result towards households with M members who are engaged in a bargaining process. Specifically, we show that we need $M+1$ observations and $M+1$ commodities to nonparametrically reject collective rationality for households with M members (with $M \geq 1$).

Next, we present the reformulation of Afriat's theorem, which is also due to Varian (1982).

Theorem 1.5. *Let $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \ldots, T\}$ be a set of observations. The following conditions are equivalent:*

(i) there exists a nonsatiated utility function that rationalizes the data;

(ii) the data satisfies GARP*;*

(iii) there exists numbers $U_j, \lambda_j > 0$, $j = 1, \ldots, T$, such that

$$\forall i, j \in \{1, \ldots, T\} : U_i - U_j \leq \lambda_j (\mathbf{p}_j)'(\mathbf{q}_i - \mathbf{q}_j);$$

(iv) there exists a concave, continuous, nonsatiated and monotonic utility function that rationalizes the data.

The numbers U_j, λ_j in part (iii) of Theorem 1.5 can be interpreted as measures of the utility and marginal utility of income at the observed demands (see Varian, 1982, for more details). By the use of these numbers, Afriat's approach offers an explicit algorithm for calculating utility functions that are consistent with the observed data.

Some remarks are in order with respect to this result. Firstly, it states that as soon as some nonsatiated utility function rationalizes the data, then there also exists a *well-behaved* utility function that captures the preferences of the individuals. [Note that the nonsatiation assumption avoids trivial rationalizations such as for example $U(\mathbf{q}) = 0$ for all $\mathbf{q}$.] This strong result does not carry over to our results for collective rationality in Chapter 2; in that chapter we need to start from concave utility functions.[3] Therefore, we only obtain analogous results for the last three statements. Relaxing the concavity assumption is work for future research.

In view of the results in the next chapters, it is also important to note that there exists several extensions of the above theorem by integrating extra assumptions concerning the preferences. A notable example is latent separability, which includes weak separability as a special case (see, e.g. Varian, 1983, Blundell and Robin, 2000, and Crawford, 2006, for inspiring results and discussion).[4] Latent separability implies that the observed commodities can be divided in groups such that (i) the preferences for each group can be represented by a well-behaved utility function and (ii) these separate utility functions can be aggregated into another well-behaved utility function. If commodities can only belong to just one group (i.e. if they are exclusive goods), then we obtain the special case of weak separability.

Clearly, this set-up is closely related to the collective approach, since we can interpret the different groups as the different household members. However, there is an important difference since for the collective models, the second condition does not hold. Indeed, the discussion of the

[3] More precisely, we need concavity in order to characterize Pareto efficiency in terms of convex utility possibility sets, which obtains the maximization program in (1.2)

[4] Another intriguing example are the characteristics models, see Browning, Blow and Crawford, 2007 for nonparametric results for an unitary model.

pseudo-Slutsky matrix in the previous section, shows that the preferences of the individual household members can no longer be aggregated. As such, the data can only satisfy the restrictions of latent separability if it also satisfy the restrictions of individual rationality. In our nonparametric setting, this implies that $GARP$ always has to be satisfied in order to satisfy latent separability, which certainly no longer holds for our results for the collective model (see Chapter 2 for more details).

Although nonparametric techniques clearly allow for more robust results (i.e. rejection is due to the basic model and not to some unverifiable functional specification that is imposed on the model), one could end up with weak results. To measure the strength of our results, one could use power measures; see, e.g., Bronars (1987), who introduced power assessment tools for the nonparametric unitary ($GARP$) condition, Andreoni and Harbaugh (2006) for a recent survey and our power measures in Chapters 3 and 4 for applications. If a model has high power, then there is a high probability that the restrictions of the model will be rejected by data that are generated by an alternative nonnested model. For example if household consumption behavior is not generated by collective rationality, then high power implies that the restrictions of the collective model are likely to be rejected. As a consequence high power models will also result in precise recovery and forecasting results. See, e.g., Blundell, Browning and Crawford (2003, 2007) for nonparametric applications and discussion in an unitary setting.

We end with two concluding remarks, concerning the power issues of the models that we present in the next chapters. Firstly, the power of our general models is rather low. So, a further line of research will be to increase the power of our tests. This could be done by restricting the general model (see, e.g. Chapters 3 and 4 for examples) or by adapting existing results for the unitary model (see, e.g., Blundell, Browning and Crawford 2003 and 2007). Secondly, a fruitful approach could also be to combine parametric and nonparametric results. That is, in a first stage, using nonparametric techniques, one derives restrictions on the functional forms that can be used; in a second stage one subsequently applies more powerful parametric techniques that satisfy these restrictions. For example, using our results in a first stage one could derive restrictions on the individual utility functions and/or bargaining weights. In a second stage, one can then incorporate these restrictions in a parametric collective model.

1.4 Conceptual analogy with production

We end this chapter by showing that the collective approach for modeling multi-person household consumption behavior can also be used to analyze the cost efficiency of multi-output production firms. In our cost efficiency analysis, we start from the basic idea that firms producing multiple outputs are benefiting from *economies of scope*; i.e. their average total cost of production decreases as a result of increasing the number of different goods produced (see, e.g., Baumol, Panzar and Willig, 1982). In our interpretation, these economies of scope originate from joint input use (e.g. the same secretary is used) and production externalities (e.g. experiences in the production of product i benefits the production of product j). Given this, we can easily introduce the conceptual analogy between the multi-person household consumption behavior and multi-output production firms.

Indeed, we can 'artificially' rephrase our general collective model for multi-person household consumption behavior as follows: the household uses inputs $\mathbf{q}$ to 'produce' utility $U^m(\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^h)$ for each individual m in the household. In this 'production process', the household benefits from public consumption (or joint input use) and externalities. The household is collectively rational if and only if it minimizes expenditures given the 'household utility' and the bargaining weights, i.e. given U^1, U^2 and μ. See also Browning and Chiappori, 1998, for more details.

If we now replace household by firm and interpret $U^m(\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^h)$ as a produced output, then we indeed obtain a multi-output production model that benefits from joint input use and externalities (and thus from economies of scope). Moreover, collective rationality then boils down to cost efficiency: if the firm is 'collectively rational' it minimizes its expenditures.

Formally, let $\mathbf{x}_j \in \mathbb{R}^n_+$ be the observed inputs; $\mathbf{p}_j \in \mathbb{R}^n_{++}$ the observed prices and $\mathbf{y}_j \in \mathbb{R}^s_+$ the observed outputs. Then we obtain the following program for the firm: each observation j is cost efficient if and only if $(\mathbf{p}_j)'\mathbf{x}_j$ equals

$$\min_{\mathbf{x} \in \mathbb{R}^n} (\mathbf{p}_j)'\mathbf{x} \text{ s.t. } \mathbf{f}(\mathbf{x}) \geq \mathbf{y}_j, \tag{1.3}$$

with $\mathbf{f}$ the production function of the firm which, as in the consumption context, depends on the weights μ_j (see Chapter 5 for a formal discussion).

The above introduced conceptual analogy between the two settings will allow us to adapt our insights of Chapter 2 to the production context. As such, by explicitly modeling the cost saving effects stemming from economies of scope, we extend the work of Varian (1984) for single output firms towards multi-output firms. Note that the bargaining weights μ also have an intuitive interpretation in the production context. They can now be interpreted as 'priority weights' which may depend on the observation at hand.

However, given the observed output information in the production context, there is an important difference between the nonparametric tests in both settings. In the consumption setting, we need the revealed preference concept to approximate the 'better than' sets, since we do not observe the utility levels of the individuals. As is clear from the minimization program described in (1.3), this is no longer the case in the production context, since now we do observe the outputs of the firm; the firm prefers individual output k of observation i (let us denote this by $< y_i >_k$) over the same output of observation j (i.e. $< y_j >_k$) if and only if $< y_i >_k \geq < y_j >_k$. So, this extra (ordering) information allows us to easily reconstruct the 'better than' sets, which in turn leads to a powerful analysis of the cost efficiency of the firm under study.

1.5 Summary and concluding remarks

In this preliminary chapter, we introduced and discussed the main ingredients for the following chapters. We motivated our choice for the collective approach for modeling multi-person household consumption behavior, by showing that it nicely deals with the criticisms on the unitary and strategic approach. Clearly, it respects the individual preferences of the household members and the Pareto efficiency assumption. But also the income pooling hypothesis is avoided, since the source of the household income may influence the bargaining weights μ_j (see, e.g, Browning and Chiappori, 1998).

Our discussion of the pseudo-Slutsky matrix clearly shows that the unitary model is a special case of the collective model. So, in order to be able to reject the restrictions of the collective model, one first has to reject the restrictions of the unitary model. We want to stress that this conclusion only holds in the absence of exclusive goods or assignable information; i.e. if we only observe the aggregate quantities and have no information concerning the intrahousehold allocation. If one has such assignable information, then the unitary and collective model are no longer nested. See, e.g., Chiappori (1988), who reaches this conclusion by the use of the observed leisure of the two members; or Cherchye, De Rock and Vermeulen (2007d).

Next, we introduced $GARP$, which is the nonparametric tool to test individual rationality (or the unitary approach). We prefer this nonparametric approach since it results in tests that do not rely on functional specifications of the individual preferences and the intrahousehold decision process. These specifications are in general not verifiable and moreover they may highly influence the outcome of the tests. Therefore, the main objective in this work will be the development of nonparametric tests of the general collective model of Browning and Chiappori (1998); as such, we generalize the nonparametric results of Varian (1982) and Chiappori (1988).

We ended the chapter by explaining the conceptual analogy between multi-member household consumption behavior and multi-output production firms. This analogy allows us to adapt the insights of the consumption setting towards the production setting. We will exploit this in Chapter 5 to obtain powerful cost efficiency tests.

Chapter 2

Collective approach to household consumption: a nonparametric characterization

Abstract

We provide a nonparametric characterization of a general collective model for M-member household consumption, which includes externalities and public consumption. Next, we establish testable necessary and sufficient conditions for data consistency with collective rationality that only include observed price and quantity information. These conditions have a structure similar to the $GARP$ for individual rationality, which is convenient from a testing point of view. In addition, we derive the minimum number of goods and observations that enable the rejection of collectively rational household behavior.[1]

[1] This chapter is adapted from Cherchye, De Rock and Vermeulen (2007a). We are grateful to a Co-Editor, three anonymous referees, Denis Beninger, Geert Dhaene and Olivier Donni for helpful comments and suggestions. We also thank seminar participants in Leuven, Mannheim, Paris, Tilburg, Turin and at the Econometric Society World Congress 2005 in London for useful discussions. Finally, we want to thank Martin Browning for inspiring conversations, which formed an important motivation for this study.

2.1 Introduction

In Chapter 1 we discussed the general collective consumption model of Browning and Chiappori (1998) and argued that it models household consumption better than the unitary model. However, the tests of Browning and Chiappori are parametric in nature and therefore rely on 'ad-hoc' functional specifications on the individual preferences and the bargaining process. Chapter 1 shows that nonparametric (or revealed preferences) tests avoid these specifications. This nonparametric approach was first adapted to the collective model by Chiappori (1988), who restricted attention to a labor supply setting that involves a number of convenient simplifications for the empirical analyst (e.g. observability of household members' leisure/labour supply and no public consumption).

In this chapter we generalize Chiappori's work (1988) by providing a nonparametric characterization of the collective consumption model of Browning and Chiappori (1998), which includes both public consumption and (*in casu* positive) externalities. This characterization allows for analyzing demand behavior without relying on (non-verifiable) functional structure that is imposed on the household decision process (i.e. the household members' preferences and the intrahousehold bargaining process). Focusing on this general model implies, however, that we can no longer use the convenient simplifications of the labor supply setting that Chiappori considered.

Section 2.2 contains this characterization, i.e. we derive necessary and sufficient nonparametric conditions for data consistency with this general model. As we will discuss, these conditions imply *unobservable* (household member-specific) quantity and price information. In Sections 2.3 and 2.4, we subsequently establish necessary and sufficient conditions that only require *observed* prices and aggregate household quantities. Interestingly, this implies nonparametric tests for collective rationality that are finite in nature, and that do not require finding a solution to a system of (nonlinear) inequalities.[2] As a by-product, we derive that the minimum number of goods and observations is at least

[2] We see at least two important differences between our approach and that of Snyder (2000), who addresses a similar research question for Chiappori's (1988) original labor supply model. First, Snyder focuses on a more restricted model that includes egoistic agents and observable leisure. Second, we do not make use of semi-algebraic theory for quantifier elimination. A well-known limitation of these latter techniques is that they become computationally cumbersome for large data

3 in order to enable a rejection of collective rationality.
Section 2.5 considers the general case with M decision makers. The last section contains some concluding remarks and the Appendix presents the proofs of our results.

2.2 A characterization of collective rationality for two-member households

As explained above, we first consider two-member (1 and 2) households. The household purchases the (non-zero) n-vector of quantities $\mathbf{q} \in \mathbb{R}^n_+$ with corresponding prices $\mathbf{p} \in \mathbb{R}^n_{++}$. All goods can be consumed privately, publicly or both. Generally, we have $\mathbf{q} = \mathbf{q}^1 + \mathbf{q}^2 + \mathbf{q}^h$ for $\mathbf{q}$ the (observed) aggregate quantities, $\mathbf{q}^1$ and $\mathbf{q}^2$ the (unobserved) private quantities of each household member, and $\mathbf{q}^h$ the (unobserved) public quantities.

Following Browning and Chiappori (1998), we consider general preferences for the household members that may depend not only on the own private and public quantities, but also (positively) on the other individual's private quantities; this allows for altruism and/or externalities.[3] Formally, this means that the preferences of each household member m ($m = 1, 2$) can be represented by a utility function of the form $U^m(\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^h)$ that is non-decreasing in its arguments $\mathbf{q}^1$, $\mathbf{q}^2$ and $\mathbf{q}^h$. Throughout, we focus on non-satiated utility functions.

Suppose T observations of the household. For each observation j we use $\mathbf{p}_j$ and $\mathbf{q}_j$ to denote the (observed) aggregate prices and quantities, respectively; while $S = \{(\mathbf{p}_j; \mathbf{q}_j), j = 1, \ldots, T\}$ represents the set of

sets. For example, Snyder restricts to settings of only two observations, while we consider the general case of T observations.

[3] This setting generalizes Chiappori's (1988) altruistic model in two ways: it does not assume the observability of private and/or public consumption of any good; and it allows for public consumption. Admittedly, the assumption of positive externalities, which is not needed in a parametric setting (see Browning and Chiappori, 1998), may be restrictive in some instances. However, its restrictive nature should not be overestimated. Even though a negative externality may be associated with e.g. tobacco consumption, the non-smoker's positive valuation of the smoker's utility generated by smoking might well outweigh that negative externality. In addition, within-household mechanisms may be instituted that decrease or even eliminate the negative externalities; see, e.g., the widespread practice of smoking outside in households consisting of smokers as well as non-smokers.

observations. For observed aggregate quantities $\mathbf{q}_j$, we define *feasible personalized quantities* $\widehat{\mathbf{q}}_j$ as

$$\widehat{\mathbf{q}}_j = (\mathfrak{q}_j^1, \mathfrak{q}_j^2, \mathfrak{q}_j^h) \text{ with } \mathfrak{q}_j^1, \mathfrak{q}_j^2, \mathfrak{q}_j^h \in \mathbb{R}_+^n \text{ and } \mathfrak{q}_j^1 + \mathfrak{q}_j^2 + \mathfrak{q}_j^h = \mathbf{q}_j. \tag{2.1}$$

Each $\widehat{\mathbf{q}}_j$ captures a feasible decomposition of the aggregate quantities $\mathbf{q}_j$ into private quantities ($\mathfrak{q}_j^1$ and $\mathfrak{q}_j^2$) and public quantities ($\mathfrak{q}_j^h$). One possible specification of these personalized quantities $\mathfrak{q}_j^1$, $\mathfrak{q}_j^2$ and $\mathfrak{q}_j^h$ are the true quantities $\mathbf{q}_j^1$, $\mathbf{q}_j^2$ and $\mathbf{q}_j^h$, but -of course- these latter quantities are not observed. Using this concept, we can now define the condition for a collective rationalization of a set of observations S.

Definition 2.1. *Let $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \ldots, T\}$ be a set of observations. A pair of utility functions U^1 and U^2 provides a* collective rationalization *of S if for each observation j there exist feasible personalized quantities $\widehat{\mathbf{q}}_j = (\mathfrak{q}_j^1, \mathfrak{q}_j^2, \mathfrak{q}_j^h)$ and $\mu_j \in \mathbb{R}_{++}$ such that*

$$U^1(\widehat{\mathbf{q}}_j) + \mu_j U^2(\widehat{\mathbf{q}}_j) \geq U^1(\widehat{\mathbf{z}}) + \mu_j U^2(\widehat{\mathbf{z}})$$

for all $\widehat{\mathbf{z}} = (\mathfrak{z}^1, \mathfrak{z}^2, \mathfrak{z}^h)$ with $\mathfrak{z}^1, \mathfrak{z}^2, \mathfrak{z}^h \in \mathbb{R}_+^n$ and $\mathbf{p}_j'(\mathfrak{z}^1 + \mathfrak{z}^2 + \mathfrak{z}^h) \leq \mathbf{p}_j'\mathbf{q}_j$.

Thus, a collective rationalization of S requires that there exists, for each observation j, feasible personalized quantities $\widehat{\mathbf{q}}_j$ that maximize a weighted sum of household member utilities U^1 and U^2 for the given household budget $\mathbf{p}_j'\mathbf{q}_j$. This optimality condition reflects the Pareto efficiency assumption regarding observed household consumption in the collective model. Each weight μ_j represents the 'bargaining power' of the household members for observation j; see Browning and Chiappori (1998) for a detailed discussion.[4]

In view of our further exposition, it is interesting to compare the collective rationality condition in Definition 2.1 with the standard unitary rationality condition. According to Varian's (1982; p. 946) definition, a *unitary rationalization* of the observed set S requires a collective rationalization with $\mu_j = 0$ and $\mathfrak{q}_j^1 = \mathbf{q}_j$ (or, equivalently, $\mathfrak{q}_j^2 = \mathfrak{q}_j^h = \mathbf{0}$) for each observation j.[5] In that presentation, unitary rationalization boils down to collective rationalization with one household member (*in casu*

[4] Given the discussion in Browning, Chiappori and Lechene (2006), we implicitly assume that the μ_j depend on observed prices and total expenditures. See also our discussion in Chapter 1.

[5] Strictly speaking, $\mu_j = 0$ is excluded in Definition 2.1. As for that definition, we note that the requirement $\mu_j \in \mathbb{R}_{++}$ pertains to the Pareto efficiency interpretation of household consumption, which is, of course, irrelevant if there is only one

member 1) as the 'dictator' in the household. This interpretation of the unitary model as a *dictatorship* model will return in our discussion in Section 2.4.

Using Definitions 1.1 and 1.3, we can now establish nonparametric conditions for a collective rationalization of a set S. To do so, we first define *feasible personalized prices* $(\widehat{\mathbf{p}}_j^1, \widehat{\mathbf{p}}_j^2)$ for observed aggregate prices $\mathbf{p}_j$, as follows:

$$\begin{array}{l} \widehat{\mathbf{p}}_j^1 = (\mathfrak{p}_j^1, \mathfrak{p}_j^2, \mathfrak{p}_j^h) \text{ and } \widehat{\mathbf{p}}_j^2 = (\mathbf{p}_j - \mathfrak{p}_j^1, \mathbf{p}_j - \mathfrak{p}_j^2, \mathbf{p}_j - \mathfrak{p}_j^h) \\ \text{with } \mathfrak{p}_j^1, \mathfrak{p}_j^2, \mathfrak{p}_j^h \in \mathbb{R}_+^n \text{ and } \mathfrak{p}_j^c \leq \mathbf{p}_j \ (c = 1, 2, h). \end{array} \tag{2.2}$$

This concept complements the concept of feasible personalized quantities in (2.1): $\widehat{\mathbf{p}}_j^1$ and $\widehat{\mathbf{p}}_j^2$ capture the fraction of the price for the personalized quantities $\widehat{\mathbf{q}}_j$ that is borne by, respectively, member 1 and member 2; $\mathfrak{p}_j^1$ and $\mathfrak{p}_j^2$ pertain to private quantities and $\mathfrak{p}_j^h$ to public quantities.[6] Based on (2.1) and (2.2), we define a *set of feasible personalized prices and quantities*

$$\widehat{S} = \{(\widehat{\mathbf{p}}_j^1, \widehat{\mathbf{p}}_j^2; \widehat{\mathbf{q}}_j); j = 1, \ldots, T\}.$$

We then have the following result.

Proposition 2.2. *Let* $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \ldots, T\}$ *be a set of observations. The following conditions are equivalent:*

(i) there exists a pair of concave and continuous utility functions U^1 *and* U^2 *that provide a* collective rationalization *of* S;

(ii) there exists a set of feasible personalized prices and quantities $\widehat{S}$ *such that the sets* $\{(\widehat{\mathbf{p}}_j^1; \widehat{\mathbf{q}}_j);\ j = 1, \ldots, T\}$ *and* $\{(\widehat{\mathbf{p}}_j^2; \widehat{\mathbf{q}}_j);\ j = 1, \ldots, T\}$ *both satisfy* GARP;

(iii) there exists a set of feasible personalized prices and quantities $\widehat{S}$, *numbers* $U_j^m > 0$ *and* $\lambda_j^m > 0$ $(m = 1, 2)$ *such that* $\forall i, j \in \{1, \ldots, T\}$: $U_i^1 - U_j^1 \leq \lambda_j^1 (\widehat{\mathbf{p}}_j^1)' (\widehat{\mathbf{q}}_i - \widehat{\mathbf{q}}_j)$ *and* $U_i^2 - U_j^2 \leq \lambda_j^2 (\widehat{\mathbf{p}}_j^2)' (\widehat{\mathbf{q}}_i - \widehat{\mathbf{q}}_j)$.

('dictator') household member. In fact, as shown in Chapter 1, unitary rationality requires a collective rationalization for μ_j constant over all observations j; but we prefer the dictatorship interpretation of the unitary model in view of our following discussion. [Compare with Browning and Chiappori, 1998; see also Browning, Chiappori and Lechene, 2006.] Further, the fact that we can use $\mathfrak{q}_j^1 = \mathbf{q}_j$ to obtain the unitary rationalization condition illustrates that the distinction between public and private consumption becomes irrelevant in the unitary model; which contrasts with the collective model.

[6] It is easily verified that $(\widehat{\mathbf{p}}_j^1 + \widehat{\mathbf{p}}_j^2)' \widehat{\mathbf{q}}_i = \mathbf{p}_j' \mathbf{q}_i$ for any i and j.

The nonparametric conditions (ii) and (iii) have a structure similar to the one for the unitary model; see Varian (1982) for an extensive discussion of the nonparametric requirements for unitary rationalization. The essential difference is that the conditions for collective rationalization are expressed in terms of a set of feasible personalized prices and quantities $\widehat{S}$. For a given specification of this set, Proposition 2.2 states nonparametric conditions *at the level of the household members 1 and 2* that are analogous to the unitary rationalization conditions *at the level of the aggregate household.* But contrary to the unitary case, the true personalized prices and quantities are unobserved. Therefore, it is only imposed that there must exist at least one $\widehat{S}$ that satisfies the conditions. Referring to our discussion in Section 1.3, also note that this result does not imply that the observed aggregate data should satisfy *GARP*, as such this model is thus not nested with the unitary model.

Proposition 2.2 states that the representation of collective rationality in Definition 2.1 can be *decentralized* in the following sense. There must exist a set $\widehat{S}$ such that the sets

$$\{(\widehat{\mathbf{p}}_j^1, \widehat{\mathbf{q}}_j); j = 1, \ldots, T\} \text{ and } \{(\widehat{\mathbf{p}}_j^2, \widehat{\mathbf{q}}_j); j = 1, \ldots, T\}$$

both satisfy the *GARP*.[7]

A final note pertains to the interpretation of the nonparametric conditions in Proposition 2.2. Following Chiappori (1988), we can interpret the different goods as 'public' goods, given that they all enter both members' utility functions. In that interpretation, the personalized prices $(\widehat{\mathbf{p}}_j^1, \widehat{\mathbf{p}}_j^2)$ can be understood as 'Lindahl prices': they must add-up (over members 1 and 2) to the observed market prices in order to be consistent with Pareto efficiency.[8] Thus, no qualitative distinction should be made between public and private quantities (where private quantities may be associated with externalities). Yet, there is a

[7] Note the similarity between the above decentralization result and the well-known decentralization result in the case of egoistic agents without public consumption; this result forms the theoretical basis for many collective rationality tests (see, e.g., Chiappori, 1988, Fortin and Lacroix, 1997, and Cherchye and Vermeulen, 2007).

[8] See, e.g., Myles (1995) for a discussion of Lindahl prices. In this respect, note that if member m has for each observation a zero Lindahl price for a given commodity, then this actually means that this commodity does not enter in his utility function. So formally, we should then adapt our notations in Definition 2.1 and the subsequent results. To keep the exposition simple and slightly abusing notation, we refrain from this.

clear quantitative difference: household members may accord another marginal valuation to private consumption than to public consumption.

2.3 Testable necessity restrictions

The (necessary and sufficient) conditions for a collective rationalization in Proposition 2.2 can be difficult to use in practice, since they are nonlinear in terms of feasible personalized prices $(\widehat{\mathbf{p}}_j^1, \widehat{\mathbf{p}}_j^2)$ and quantities $\widehat{\mathbf{q}}_j$; they are namely defined in products of both concepts. See, e.g., Watson, Bartholomew-Biggs and Ford (2000) for a discussion of similar nonlinearity problems. In the following, we present testable conditions for collective rationality that solely use (observed) aggregate prices $\mathbf{p}_j$ and quantities $\mathbf{q}_j$. This section develops a necessary condition for a collective rationalization of a set of observations S that has a similar two-step structure as the unitary *GARP* (see our discussion following Definition 1.3). The next section presents a complementary sufficiency condition.
We first define the analogues of the relations R_0 and R for members 1 and 2 in the collective model.

Definition 2.3. *Let $\widehat{S} = \{(\widehat{\mathbf{p}}_j^1, \widehat{\mathbf{p}}_j^2; \widehat{\mathbf{q}}_j); j = 1, \ldots, T\}$ be a set of feasible personalized prices and quantities. Then for $m = 1, 2$: if $(\widehat{\mathbf{p}}_i^m)'\widehat{\mathbf{q}}_i \geq (\widehat{\mathbf{p}}_i^m)'\widehat{\mathbf{q}}_j$ then $\widehat{\mathbf{q}}_i \; R_0^m \; \widehat{\mathbf{q}}_j$; and if $\widehat{\mathbf{q}}_i \; R_0^m \; \widehat{\mathbf{q}}_k$, $\widehat{\mathbf{q}}_k \; R_0^m \; \widehat{\mathbf{q}}_l$, $\ldots$, $\widehat{\mathbf{q}}_z \; R_0^m \; \widehat{\mathbf{q}}_j$ for some (possibly empty) sequence $(k, l, \ldots, z)$ then $\widehat{\mathbf{q}}_i \; R^m \; \widehat{\mathbf{q}}_j$.*

Of course, different specifications of the set $\widehat{S}$ generally imply different relations R_0^m and R^m. To establish our testable necessary condition for collectively rational behavior, we derive restrictions on the relations R_0^m and R^m without reference to a specific $\widehat{S}$. In this respect, the next lemma specifies a useful relationship between R_0^m and R_0, which is defined in terms of the set of observations S.

Lemma 2.4. *Let $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \ldots, T\}$ be a set of observations. We have $\mathbf{q}_i \; R_0 \; \mathbf{q}_j$ if and only if, for all sets $\widehat{S}$ of feasible personalized prices and quantities, $\widehat{\mathbf{q}}_i \; R_0^1 \; \widehat{\mathbf{q}}_j$ or $\widehat{\mathbf{q}}_i \; R_0^2 \; \widehat{\mathbf{q}}_j$.*

The intuition of this result pertains to the Pareto efficient nature of household behavior in the collective model. Specifically, if the household has chosen $\mathbf{q}_i$ when $\mathbf{q}_j$ was equally available (i.e. $\mathbf{q}_i \; R_0 \; \mathbf{q}_j$, which means $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$), then we always have that, *independently of the specification of the set* $\widehat{S}$, at least one household member must prefer the former (personalized) quantities to the latter (i.e. $\widehat{\mathbf{q}}_i \ R_0^1 \ \widehat{\mathbf{q}}_j$ or $\widehat{\mathbf{q}}_i \ R_0^2 \ \widehat{\mathbf{q}}_j$). As a result, if we want to avoid selecting specific feasible personalized prices and quantities -because we lack information to do so-, then we can start from the relation R_0 for specifying restrictions on the relations R_0^1 and R_0^2. Moreover, the equivalence result in Lemma 2.4 implies that we cannot do better when only using the set of observations S (rather than some $\widehat{S}$).

Lemma 2.4 provides the starting point for our testable necessity condition for collective rationality. We sketch the basic intuition of that condition by means of the next simple example.

Example 2.5. *Consider the case of three observations and three goods with prices and quantities*

$$\mathbf{q}_1 = (\,8\;2\;1\,)', \mathbf{q}_2 = (\,2\;1\;8\,)', \mathbf{q}_3 = (\,1\;8\;2\,)';$$
$$\mathbf{p}_1 = (\,5\;2\;1\,)', \mathbf{p}_2 = (\,2\;1\;5\,)', \mathbf{p}_3 = (\,1\;5\;2\,)'.$$

This specific data structure implies that

$$\mathbf{p}_1'\mathbf{q}_1 > \mathbf{p}_1'(\mathbf{q}_2 + \mathbf{q}_3),\ \mathbf{p}_2'\mathbf{q}_2 > \mathbf{p}_2'(\mathbf{q}_1 + \mathbf{q}_3) \text{ and } \mathbf{p}_3'\mathbf{q}_3 > \mathbf{p}_3'(\mathbf{q}_1 + \mathbf{q}_2),$$

so that for all observations $i, j \in \{1, 2, 3\}$ we have $\mathbf{q}_i \ R_0 \ \mathbf{q}_j$. Using Lemma 2.4, we therefore conclude

$$\forall i, j \in \{1, 2, 3\} : \widehat{\mathbf{q}}_i \ R_0^1 \ \widehat{\mathbf{q}}_j \text{ or } \widehat{\mathbf{q}}_i \ R_0^2 \ \widehat{\mathbf{q}}_j. \tag{2.3}$$

Given this, one possible specification of the relations R_0^1 and R_0^2 is

$$\widehat{\mathbf{q}}_1 \ R_0^1 \ \widehat{\mathbf{q}}_2,\ \widehat{\mathbf{q}}_2 \ R_0^1 \ \widehat{\mathbf{q}}_3 \text{ and } \widehat{\mathbf{q}}_3 \ R_0^2 \ \widehat{\mathbf{q}}_2,\ \widehat{\mathbf{q}}_2 \ R_0^2 \ \widehat{\mathbf{q}}_1. \tag{2.4}$$

Intuitively, this specification means that member 1 prefers (personalized) $\widehat{\mathbf{q}}_1$ over $\widehat{\mathbf{q}}_2$ while member 2 prefers $\widehat{\mathbf{q}}_3$ over $\widehat{\mathbf{q}}_2$. In that case, the choice of the (aggregate) quantities $\mathbf{q}_2$ can be rationalized only if it is not more expensive than the sum of $\mathbf{q}_1$ and $\mathbf{q}_3$, which requires that $\mathbf{p}_2'\mathbf{q}_2 \leq \mathbf{p}_2'(\mathbf{q}_1 + \mathbf{q}_3)$. But this is inconsistent with $\mathbf{p}_2'\mathbf{q}_2 > \mathbf{p}_2'(\mathbf{q}_1 + \mathbf{q}_3)$. Because the same argument can be repeated for any other possible specification of the relations R_0^1 and R_0^2 instead of (2.4), we conclude that a collective rationalization of this set of observations is impossible.[9]

[9] At this point, it is important that we can exclude for all $i, j \in \{1, 2, 3\}$ with $i \neq j$: $\widehat{\mathbf{q}}_i \ R_0^1 \ \widehat{\mathbf{q}}_j$ *and* $\widehat{\mathbf{q}}_i \ R_0^2 \ \widehat{\mathbf{q}}_j$. Intuitively, the latter specification of the relations R_0^1 and R_0^2 means that both members 1 and 2 prefer (personalized) $\widehat{\mathbf{q}}_i$ over $\widehat{\mathbf{q}}_j$. In that case, the choice of (aggregate) $\mathbf{q}_j$ can be rationalized only if it is not more expensive than $\mathbf{q}_i$, which is inconsistent with $\mathbf{p}_j'\mathbf{q}_j > \mathbf{p}_j'\mathbf{q}_i$. The formal argument is based on Lemma 2.6 (rule (iv)).

The basic structure of the collective rationalization test in this example parallels the two-step structure of the unitary *GARP* test. Specifically, we first specified the relations R_0^1 and R_0^2 in (2.4), and subsequently verified the corresponding upper cost bound condition (*in casu* $\mathbf{p}_2'\mathbf{q}_2 \leq \mathbf{p}_2'(\mathbf{q}_1 + \mathbf{q}_3)$), which is not met for this particular set of observations.

To generalize these ideas, we first specify some further restrictions that must hold if a collective rationalization of the set of observations S is possible in terms of Proposition 2.2. In that case, there exists a set of feasible personalized prices and quantities $\widehat{S}$ such that the corresponding R_0^1 and R_0^2 satisfy the following conditions in relation to their transitive closures R^1 and R^2 and the aggregate prices $\mathbf{p}_j$ and quantities $\mathbf{q}_j$.

Lemma 2.6. *Suppose that there exists a pair of utility functions U^1 and U^2 that provide a* collective rationalization *of the set of observations $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \ldots, T\}$. Then there exists a set of feasible personalized prices and quantities $\widehat{S}$ that defines the relations R_0^m, R^m for each member $m \in \{1, 2\}$ such that:*

(i) if $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$ and $\widehat{\mathbf{q}}_j \; R^m \; \widehat{\mathbf{q}}_i$ then $\widehat{\mathbf{q}}_i \; R_0^l \; \widehat{\mathbf{q}}_j$ (with $m \neq l$);

(ii) if $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'(\mathbf{q}_{j_1} + \mathbf{q}_{j_2})$ and $\widehat{\mathbf{q}}_{j_1} \; R^m \; \widehat{\mathbf{q}}_i$ then $\widehat{\mathbf{q}}_i \; R_0^l \; \widehat{\mathbf{q}}_{j_2}$ (with $m \neq l$);

(iii) if $\widehat{\mathbf{q}}_{i_1} \; R^1 \; \widehat{\mathbf{q}}_j$ and $\widehat{\mathbf{q}}_{i_2} \; R^2 \; \widehat{\mathbf{q}}_j$ then $\mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'(\mathbf{q}_{i_1} + \mathbf{q}_{i_2})$;

(iv) if $\widehat{\mathbf{q}}_i \; R^1 \; \widehat{\mathbf{q}}_j$ and $\widehat{\mathbf{q}}_i \; R^2 \; \widehat{\mathbf{q}}_j$ then $\mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'\mathbf{q}_i$.

The interpretation of this result pertains to the very nature of the collective model, which -to recall- explicitly recognizes the multi-person nature of the household decision process. More specifically, the four rules in Lemma 2.6 relate to *rationality across household members* for a given specification of the feasible personalized prices and quantities. First, rule (i) expresses that, if member m prefers (personalized) $\widehat{\mathbf{q}}_j$ over $\widehat{\mathbf{q}}_i$ for (aggregate) $\mathbf{q}_j$ not more expensive than $\mathbf{q}_i$, then the choice of $\mathbf{q}_i$ can be rationalized only if the other member l prefers $\widehat{\mathbf{q}}_i$ over $\widehat{\mathbf{q}}_j$. Next, the meaning of rule (ii) is that, if (aggregate) $\mathbf{q}_i$ is more expensive than the sum of $\mathbf{q}_{j_1}$ and $\mathbf{q}_{j_2}$, while member m prefers (personalized) $\widehat{\mathbf{q}}_{j_1}$ over $\widehat{\mathbf{q}}_i$, then the only possibility for rationalizing the choice of $\mathbf{q}_i$ is that the other member l prefers $\widehat{\mathbf{q}}_i$ over $\widehat{\mathbf{q}}_{j_2}$.

Rules (i) and (ii) define restrictions on the relations R_0^m and R^m. For a specification of these relations, rules (iii) and (iv) define the corresponding upper cost bound conditions. First, rule (iii) complements rule (ii): if members 1 and 2 prefer respectively (personalized) $\widehat{\mathbf{q}}_{i_1}$ and

$\widehat{\mathbf{q}}_{i_2}$ over $\widehat{\mathbf{q}}_j$, then the choice of (aggregate) $\mathbf{q}_j$ can be rationalized only if it is not more expensive than the sum of $\mathbf{q}_{i_1}$ and $\mathbf{q}_{i_2}$. Finally, rule (iv) considers the special case where both members prefer the same (personalized) quantities $\widehat{\mathbf{q}}_i$ over $\widehat{\mathbf{q}}_j$; in that case, under the prices $\mathbf{p}_j$ the quantities $\mathbf{q}_j$ cannot be associated with a strictly higher expenditure level than $\mathbf{q}_i$.

Lemma 2.6 states that, if a collective rationalization of the set of observations S is possible, then *there exists* a set of feasible personalized prices and quantities $\widehat{S}$ that is consistent with the rules (i)-(iv). To recall, Lemma 2.4 states that, if $\mathbf{q}_i\ R_0\ \mathbf{q}_j$ (or, equivalently, $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$), then *for any* specification of the set $\widehat{S}$ we must have $\widehat{\mathbf{q}}_i\ R_0^1\ \widehat{\mathbf{q}}_j$ or $\widehat{\mathbf{q}}_i\ R_0^2\ \widehat{\mathbf{q}}_j$. That is,

$$\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j \Rightarrow \widehat{\mathbf{q}}_i\ R_0^1\ \widehat{\mathbf{q}}_j \text{ or } \widehat{\mathbf{q}}_i\ R_0^2\ \widehat{\mathbf{q}}_j. \tag{2.5}$$

Using this, we can specify restrictions on the relations R_0^1 and R_0^2 in terms of the set of observations S, i.e. without explicit reference to a set of feasible personalized prices and quantities $\widehat{S}$. If there does not exist a specification of the relations R_0^1 and R_0^2, and corresponding transitive closures R^1 and R^2, that are consistent with (2.5) and at the same time meet rules (i)-(iv) in Lemma 2.6, then a collective rationalization of the set of observations S is impossible. Or, a necessary condition for a collective rationalization of the set S to be possible is that there exists a specification of R_0^m and R^m $(m = 1, 2)$ that is consistent with (2.5) and rules (i)-(iv) in Lemma 2.6. This idea underlies our testable necessity condition for collective rationality that is expressed directly in terms of the set of observations S of aggregate prices and quantities; the condition essentially combines the results in Lemmas 2.4 and 2.6.

To formalize the idea, we introduce some additional notation. First, referring to (2.5), for $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$ we use $\mathbf{q}_i\ H_0^1\ \mathbf{q}_j$ if we *hypothesize* $\widehat{\mathbf{q}}_i\ R_0^1\ \widehat{\mathbf{q}}_j$ and $\mathbf{q}_i\ H_0^2\ \mathbf{q}_j$ if we *hypothesize* $\widehat{\mathbf{q}}_i\ R_0^2\ \widehat{\mathbf{q}}_j$; let H^1 and H^2 denote the transitive closures of these *hypothetical* relations H_0^1 and H_0^2. The existence of a set of feasible personalized prices and quantities $\widehat{S}$ that satisfies the conditions in Proposition 2.2 implies that there exist relations H_0^m and H^m consistent with the analogues of rules (i)-(iv) in Lemma 2.6.

Proposition 2.7. *Suppose that there exists a pair of utility functions U^1 and U^2 that provide a* collective rationalization *of the set of observations $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \ldots, T\}$. Then there exist* hypothetical *relations H_0^m, H^m for each member $m \in \{1, 2\}$ such that:*

(i) if $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$ *then* $\mathbf{q}_i\ H_0^1\ \mathbf{q}_j$ *or* $\mathbf{q}_i\ H_0^2\ \mathbf{q}_j$;

(ii) if $\mathbf{q}_i\ H_0^m\ \mathbf{q}_k$, $\mathbf{q}_k\ H_0^m\ \mathbf{q}_l$, $\ldots$, $\mathbf{q}_z\ H_0^m\ \mathbf{q}_j$ *for some (possibly empty) sequence* $(k, l, \ldots, z)$ *then* $\mathbf{q}_i\ H^m\ \mathbf{q}_j$;

(iii) if $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$ *and* $\mathbf{q}_j\ H^m\ \mathbf{q}_i$ *then* $\mathbf{q}_i\ H_0^l\ \mathbf{q}_j$ *(with* $m \neq l$*)*;

(iv) if $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'(\mathbf{q}_{j_1}+\mathbf{q}_{j_2})$ *and* $\mathbf{q}_{j_1}\ H^m\ \mathbf{q}_i$ *then* $\mathbf{q}_i\ H_0^l\ \mathbf{q}_{j_2}$ *(with* $m \neq l$*)*;

(v) if $\mathbf{q}_{i_1}\ H^1\ \mathbf{q}_j$ *and* $\mathbf{q}_{i_2}\ H^2\ \mathbf{q}_j$ *then* $\mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'(\mathbf{q}_{i_1}+\mathbf{q}_{i_2})$;

(vi) if $\mathbf{q}_i\ H^1\ \mathbf{q}_j$ *and* $\mathbf{q}_i\ H^2\ \mathbf{q}_j$ *then* $\mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'\mathbf{q}_i$.

The intuition of the different rules follows immediately from our discussion of Lemmas 2.4 and 2.6, when replacing the relations R_0^m and R^m by their hypothetical counterparts H_0^m and H^m. More specifically, rule (i) refers to the result in Lemma 2.4. Rule (ii) defines the transitive closures H^1 and H^2 of the relations H_0^1 and H_0^2; compare with Definition 2.3. Finally, rules (iii)-(vi) comply with rules (i)-(iv) in Lemma 2.6.

To illustrate the proposition, we recapture our Example 2.5.

Example 2.5 (continued). *The first step of our argument in Example 2.5 pertains to rule (i) in Proposition 2.7. Specifically, we can rephrase (2.3) in terms of the hypothetical relations* H_0^1 *and* H_0^2 *as*

$$\forall i, j \in \{1,2,3\} : \mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j \ \Rightarrow \mathbf{q}_i\ H_0^1\ \mathbf{q}_j \text{ or } \mathbf{q}_i\ H_0^2\ \mathbf{q}_j.$$

Similarly, (2.4) complies with

$$\mathbf{q}_1\ H_0^1\ \mathbf{q}_2,\ \mathbf{q}_2\ H_0^1\ \mathbf{q}_3 \text{ and } \mathbf{q}_3\ H_0^2\ \mathbf{q}_2,\ \mathbf{q}_2\ H_0^2\ \mathbf{q}_1.$$

Rule (v) in Proposition 2.7 then requires $\mathbf{p}_2'\mathbf{q}_2 \leq \mathbf{p}_2'(\mathbf{q}_1+\mathbf{q}_3)$*, and this upper cost bound condition is not met by this set of observations. A similar inconsistency result holds for any other specification of the hypothetical relations* $H_0^m, H^m (m = 1, 2)$*: one can verify that any such specification that is consistent with rules (i)-(iv) cannot meet the corresponding upper cost bound conditions (v)-(vi).*

Interestingly, Example 2.5 implies that it is sufficient to have three goods and three observations for rejecting collective rationality of observed household behavior. The following proposition states that this is also necessary.

Proposition 2.8. *There do not always exist utility functions* U^1 *and* U^2 *that provide a* collective rationalization *of the set of observations* $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \ldots, T\}$ *if and only if (i) the number of goods* $n \geq 3$ *and (ii) the number of observations* $T \geq 3$.

We only sketch the basic idea for the necessity result.[10] First, consider that are only two goods ($n = 2$) and T (≥ 2) observations. In that case, a collective rationalization of the set of observations S is always achieved for the following specification of feasible personalized prices and quantities (for $(\mathbf{x})_e$ the e-th entry of the vector $\mathbf{x}$):

$$\forall j : \mathfrak{p}_j^1 = \mathbf{p}_j \text{ and } \mathfrak{p}_j^2 = \mathfrak{p}_j^h = \mathbf{0};\ (\mathfrak{q}_j^1)_1 = (\mathbf{q}_j)_1 \text{ and } (\mathfrak{q}_j^2)_2 = (\mathbf{q}_j)_2.$$

In words, goods 1 and 2 are allocated exclusively to, respectively, member 1 and member 2; for each observation j we have $(\widehat{\mathbf{p}}_j^1)'\widehat{\mathbf{q}}_j = (\mathbf{p}_j)_1(\mathbf{q}_j)_1$ and $(\widehat{\mathbf{p}}_j^2)'\widehat{\mathbf{q}}_j = (\mathbf{p}_j)_2(\mathbf{q}_j)_2$. As in Chapter 1, since we have to deal product of scalars, it is easily verified that this specification of the feasible personalized quantities obtains consistency with the non-parametric conditions (ii) in Proposition 2.2.

Next, consider that there are only two observations ($T = 2$) and n (≥ 2) goods. In that case, a collective rationalization of the set of observations S is always achieved for

$$\begin{aligned}
&\mathfrak{p}_j^1 = \mathbf{p}_j \text{ and } \mathfrak{p}_j^2 = \mathfrak{p}_j^h = \mathbf{0} \text{ for } j = 1, 2;\\
&\mathfrak{q}_1^1 = \mathbf{q}_1 \text{ (or } \mathfrak{q}_1^2 = \mathfrak{q}_1^h = \mathbf{0}) \text{ and } \mathfrak{q}_2^2 = \mathbf{q}_2 \text{ (or } \mathfrak{q}_2^1 = \mathfrak{q}_2^h = \mathbf{0}).
\end{aligned}$$

In words, members 1 and 2 are the 'dictators' in, respectively, observation 1 (as $\mathfrak{q}_1^1 = \mathbf{q}_1$ and $(\widehat{\mathbf{p}}_1^1)'\widehat{\mathbf{q}}_1 = \mathbf{p}_1'\mathbf{q}_1$) and observation 2 (as $\mathfrak{q}_2^2 = \mathbf{q}_2$ and $(\widehat{\mathbf{p}}_2^2)'\widehat{\mathbf{q}}_2 = \mathbf{p}_2'\mathbf{q}_2$). Again, it is easy to verify consistency with conditions (ii) in Proposition 2.2 for this specification of the feasible personalized prices and quantities.

Thus, the collective model can be rejected (or empirical testing is meaningful) as soon as there are at least three goods and three observations. Note that the lower bound of three goods is below the lower bound derived by Browning and Chiappori (1998) in their parametric setting: empirical falsification of their collective model necessitates at least five goods. The reason is that these authors focus on pseudo-Slutsky symmetry, which requires at least five goods for testable implications. By contrast, their parametric model equally needs only three goods for testing pseudo-Slutsky negativity.[11]

[10] The following arguments concentrate on $n = 2$ (for $T \geq 2$) and on $T = 2$ (for $n \geq 2$); if the necessity result holds in these cases, then it certainly also holds for $n < 2$ and $T < 2$.

[11] We are grateful to an anonymous referee for pointing this out.

To conclude, since the necessary condition in Proposition 2.7 only requires aggregate prices $\mathbf{p}_j$ and quantities $\mathbf{q}_j$, it enables an operational collective rationality test that applies to the general case of T observations. Chapter 3 presents a finite algorithm for verifying the condition, and contains some further discussion regarding the practicality of the approach.

2.4 Testable sufficiency restrictions

While the condition in Proposition 2.7 is necessary for a collective rationalization, it is in general not sufficient.[12] This follows from Example 2.9 below, which contains data that satisfy the condition but cannot be collectively rationalized in the sense of Proposition 2.2.

Example 2.9. *We prove in the Appendix that a collective rationalization cannot be obtained for a set of seven observations with:*

$$\begin{aligned}
&\forall i \in \{1,\ldots,7\} : \mathbf{p}_i'\mathbf{q}_i > \mathbf{p}_i'\mathbf{q}_j, \forall j \in \{1,\ldots,7\}\backslash\{i\};\\
&\forall i \in \{1,7\} : \mathbf{p}_i'\mathbf{q}_i > \mathbf{p}_i'(\mathbf{q}_j + \mathbf{q}_k), \forall j,k \in \{1,\ldots,7\}\backslash\{i\}, j \neq k;\\
&\forall i \in \{2,\ldots,6\} : \mathbf{p}_i'\mathbf{q}_i = \mathbf{p}_i'(\mathbf{q}_j + \mathbf{q}_k) - \varepsilon, \forall j,k \in \{1,\ldots,7\}\backslash\{i\}, j \neq k;
\end{aligned}$$

where $\frac{\min\limits_{i,e}(\mathbf{p}_i)_e \min\limits_{i,e}(\mathbf{q}_i)_e}{6} > \varepsilon > 0$ *(*$i \in \{1,\ldots,7\}$ *and* $e \in \{1,\ldots,n\}$*).*
For example, such a structure applies to $\mathbf{q}_i$, $\mathbf{p}_i \in \mathbb{R}^7$ *with*

$$\begin{aligned}
&\forall i \in \{1,\ldots,7\} : (\mathbf{q}_i)_i = 3 \text{ and } (\mathbf{q}_i)_e = 1 \text{ if } e \neq i,\\
&\forall i \in \{1,7\} : (\mathbf{p}_i)_i = 11 \text{ and } (\mathbf{p}_i)_e = 1 \text{ if } e \neq i, \text{ and}\\
&\forall i \in \{2,\ldots,6\} : (\mathbf{p}_i)_i = 10 - \varepsilon \text{ and } (\mathbf{p}_i)_e = 1 \text{ if } e \neq i,
\end{aligned}$$

where $(1/6) > \varepsilon > 0$.

We next present a sufficient condition for a collective rationalization that solely uses observed (aggregate) prices and quantities. Essentially, as compared to the necessary condition in Proposition 2.7, this sufficient condition requires some additional structure in these prices and quantities, so that we can *always* conceive a household decision model

[12] In fact, it can be verified that the necessary condition in Proposition 2.7 is also sufficient for $T \leq 4$ (for compactness, we abstract from a formal statement). While Example 2.9 uses $T = 7$ for mathematical elegance of the proof, it is worth stressing that similar (but less elegant) arguments can be established for $4 < T < 7$.

(and corresponding feasible personalized prices and quantities) consistent with the collective rationality restrictions in Proposition 2.2; we explain the particular decision model below. Like before, this condition implies (*in casu* sufficiency) tests for collective rationality that hold for the general case of T observations. Again a finite testing algorithm is presented in the Chapter 3.

Proposition 2.10. *Let $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \ldots, T\}$ be a set of observations. Suppose that there exist* hypothetical *relations H_0^m, H^m for each member $m \in \{1, 2\}$ that satisfy rules (i)-(vi) in Proposition 2.7 and in addition allow for constructing sets S^1 and S^2 with $S^1 \subseteq S$ and $S^2 = S \backslash S^1$ such that*

(vii) $S^m = \{(\mathbf{p}_j; \mathbf{q}_j) \in S \mid \mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'\mathbf{q}_i$ *whenever* $\mathbf{q}_i\ H^m\ \mathbf{q}_j\}$*;*

(viii) for each $(\mathbf{p}_i; \mathbf{q}_i)$, $(\mathbf{p}_j; \mathbf{q}_j) \in S^m$: $\mathbf{q}_i H_0^m \mathbf{q}_j$ *whenever* $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$.

Then there exists a pair of utility functions U^1 and U^2 that provide a collective rationalization *of the set S.*

Referring to the interpretation of the unitary model as a dictatorship model (see Section 2.2), we can interpret this result in terms of a *situation-dependent dictatorship* model. Specifically, we prove in the Appendix that under conditions (i)-(viii) we can obtain consistency with the nonparametric condition (ii) in Proposition 2.2 for the following specification of the feasible personalized quantities and prices:

$$\begin{aligned}
&\text{if } (\mathbf{p}_j; \mathbf{q}_j) \in S^1 \text{ then } \mathfrak{q}_j^1 = \mathbf{q}_j, \\
&\text{if } (\mathbf{p}_j; \mathbf{q}_j) \in S^2 \text{ then } \mathfrak{q}_j^2 = \mathbf{q}_j, \\
&\text{and } \mathfrak{p}_j^1 = \mathbf{p}_j,\ \mathfrak{p}_j^2 = \mathfrak{p}_j^h = \mathbf{0} \text{ for all } (\mathbf{p}_j; \mathbf{q}_j) \in S.
\end{aligned}$$

For all observations j such that $(\mathbf{p}_j; \mathbf{q}_j) \in S^1$, member 1 is the 'dictator' because $\mathfrak{q}_j^1 = \mathbf{q}_j$ (or, equivalently, $\mathfrak{q}_j^2 = \mathfrak{q}_j^h = \mathbf{0}$) and $(\widehat{\mathbf{p}}_j^1)'\widehat{\mathbf{q}}_j = \mathbf{p}_j'\mathbf{q}_j$. Similarly, member 2 is the dictator for the other observations.[13] Or put another way, *the identity of the dictator depends on the observation*

[13] We note that, technically, this specification of the feasible personalized quantities and prices is consistent with $\infty > \mu_j > 0$ for all j (see the Appendix for details). An interpretation in terms of bargaining power is as follows (for the given specification of the personalized prices): for $(\mathbf{p}_j; \mathbf{q}_j) \in S^1$ the value of the bargaining weight μ_j (> 0) of member 2 is too small for obtaining $\mathfrak{q}_j^1 \neq \mathbf{q}_j$; and, conversely, for $(\mathbf{p}_j; \mathbf{q}_j) \in S^2$ the value of μ_j ($< \infty$) is too large for $\mathfrak{q}_j^2 \neq \mathbf{q}_j$. Further, we stress that the given specification of the feasible personalized prices and quantities should not be the unique one that obtains consistency with condition (ii) in Proposition 2.2 (and, thus, other interpretations of the sufficiency result are equally possible).

or situation at hand. In that interpretation, the statement $\mathbf{q}_i\ H^1\ \mathbf{q}_j$ means that the (situation-dependent) dictator 1 prefers the (aggregate) $\mathbf{q}_i$ over $\mathbf{q}_j$; a directly similar interpretation holds for $\mathbf{q}_i\ H^2\ \mathbf{q}_j$. Rule (vii) then specifies that the situation-dependent dictators 1 and 2 must respect the corresponding upper cost bounds. The additional rule (viii) indicates that, if member m (1 or 2) is the dictator in situations i and j, then the choice of $\mathbf{q}_i$ when $\mathbf{q}_j$ was equally obtainable under the prices $\mathbf{p}_i$ can be rationalized only if member m prefers (aggregate) $\mathbf{q}_i$ over $\mathbf{q}_j$ (or $\mathbf{q}_i\ H_0^m\ \mathbf{q}_j$).

This situation-dependent dictatorship model can be regarded as a direct 'collective' extension of the unitary decision model. Specifically, in contrast to the latter model, the former model implies *two* separate decision-makers in the household, who are each (*fully*) responsible for a disjoint subset of the T observed aggregate quantities. Consequently, the sufficiency condition implies that there must exist a partitioning of the observed set S in two subsets that each individually meet the unitary *GARP*; i.e. each individual dictator must act consistent with the unitary rationality condition *for those quantities for which she or he is (fully) responsible*. It is this interpretation that underlies the testing algorithm in the next chapter.[14]

In summary, violation of the necessary condition in Proposition 2.7 means that a collective rationalization is impossible, while consistency with the sufficient condition in Proposition 2.10 entails the opposite conclusion. As for data that meet the necessity but not the sufficiency condition, we cannot directly tell from the observed (aggregate) prices and quantities whether a collective rationalization of the data is effectively possible.[15] For instance, the proof of the inconsistency result in

[14] At this point it is interesting to note that Proposition 2.10 need not be the only sufficient condition. For instance, situation dependent dictators could be responsible for sets of commodities instead of sets of whole bundles. However this will lead to more time consuming algorithms. We thank I. Crawford for pointing this out.

[15] At this point, it is worth emphasizing the subtle difference between 'collective rationality of household behavior' and 'a collective rationalization of a set of household observations S'. On the one hand, impossibility of a collective rationalization of S (e.g. inconsistency with the necessity condition in Proposition 2.7) necessarily implies collectively irrational behavior. On the other hand, possibility of a collective rationalization of S (e.g. consistency with the sufficiency condition in Proposition 2.10) does not necessarily imply collectively rational behavior; it only means that we cannot reject collective rationality on the basis of the available set of observations.

Example 2.9 starts from the necessity condition (which, like the unitary *GARP*, focuses on the full consumption bundles), to subsequently consider the construction of feasible personalized prices and quantities for individual goods. Such practice generally boils down to checking the inequalities in Proposition 2.2 that are nonlinear in these feasible personalized prices and quantities. [We avoid this in our proof of the result in Example 2.9 only because of our specific condition for ε.]

Still, even though the necessary condition should not generally coincide with the sufficient condition, we may expect the two conditions to become equally powerful (or 'converge') when the sample size increases.[16] Specifically, if T gets larger, we have for each observation j that $\min\limits_{\mathbf{q}_i}\{\mathbf{p}_j'\mathbf{q}_i \mid \mathbf{q}_i\ H^1\ \mathbf{q}_j$ and not $\mathbf{q}_i\ H^2\ \mathbf{q}_j\}$ or $\min\limits_{\mathbf{q}_i}\{\mathbf{p}_j'\mathbf{q}_i \mid \mathbf{q}_i\ H^2\ \mathbf{q}_j$ and not $\mathbf{q}_i\ H^1\ \mathbf{q}_j\}$ will generally get closer to zero (since a larger sample can induce extra price-quantity variation). Hence, the requirement $\mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'(\mathbf{q}_{i_1} + \mathbf{q}_{i_2})$ whenever $\mathbf{q}_{i_1}\ H^1\ \mathbf{q}_j$ and $\mathbf{q}_{i_2}\ H^2\ \mathbf{q}_j$ in Proposition 2.7 (rule (v)) will approach the condition $\mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'\mathbf{q}_i$ whenever $\mathbf{q}_i\ H^m\ \mathbf{q}_j$ for $m = 1$ or 2 in Proposition 2.10 (rule (vii)).[17]

The associated 'convergence rate' will then of course depend (positively) upon the variation in the observed prices and quantities, and hence we may expect it to increase with the number of goods. For a given number of goods, the speed of convergence will vary with the specific data generating process that underlies the aggregate prices and quantities, which in turn depends on the household member utilities and on the characteristics of the within-household bargaining process. But, in general, we can safely argue that the empirical implications of the fairly rudimentary situation-dependent dictatorship solution (see the sufficient condition) will get closer to those of any more refined intrahousehold decision process (see the necessary condition) when the sample size increases.

[16] See, e.g., Bronars (1987) for power notions in the context of nonparametric rationality tests.

[17] Note that the necessary condition (rule (vi)) and the sufficient condition (rule (vii)) both require $\mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'\mathbf{q}_i$ whenever $\mathbf{q}_i H^1 \mathbf{q}_j$ *and* $\mathbf{q}_i H^2 \mathbf{q}_j$. Also observe that the empirical restrictions following from rule (iv) in Proposition 2.7 imply those of rule (viii) in Proposition 2.10 when, for each observation j, $\min\limits_{\mathbf{q}_i}\{\mathbf{p}_j'\mathbf{q}_i \mid \mathbf{q}_i H^1 \mathbf{q}_j$ and not $\mathbf{q}_i H^2 \mathbf{q}_j\}$ or $\min\limits_{\mathbf{q}_i}\{\mathbf{p}_j'\mathbf{q}_i \mid \mathbf{q}_i H^2 \mathbf{q}_j$ and not $\mathbf{q}_i H^1 \mathbf{q}_j\}$ gets close to zero for large T.

2.5 Generalization towards M members

This section generalizes our main results for two-member households towards M-member households (or more general groups): Subsection 2.5.1 contains the nonparametric characterization of collective rationality, Subsection 2.5.2 discusses the minimum number of goods and observations that enable rejection of M-member collective rationality, and Subsection 2.5.3 presents the testable collective rationality conditions. Note that this general case includes the two-member model ($M = 2$) and the unitary model ($M = 1$) as special cases.

2.5.1 A characterization of collective rationality for M-member households

The household's observed aggregate quantities $\mathbf{q}$ are now decomposed into M quantities $\mathbf{q}^m$ $(m = 1, \ldots, M)$ that capture private consumption and quantities $\mathbf{q}^h$ that represent public consumption. The different quantities are interrelated as follows:

$$\mathbf{q} = \mathbf{q}^1 + \mathbf{q}^2 + \ldots + \mathbf{q}^M + \mathbf{q}^h.$$

Each member m is further characterized by own preferences that are represented by a non-satiated utility function $U^m(\mathbf{q}^1, \mathbf{q}^2, \ldots, \mathbf{q}^M, \mathbf{q}^h)$ that is non-decreasing in its arguments.

As was the case for two-member households, we assume a set of T observations of prices and quantities; where $S = \{(\mathbf{p}_j; \mathbf{q}_j),\ j = 1, \ldots, T\}$ denotes the set of observations. To generalize Definition 2.1 of Section 2.2, for observed quantities $\mathbf{q}_j$, we define *feasible personalized quantities* $\widehat{\mathbf{q}}_j$ as

$$\begin{aligned}&\widehat{\mathbf{q}}_j = (\mathfrak{q}_j^1, \ldots, \mathfrak{q}_j^M, \mathfrak{q}_j^h)\\ &\text{with } \mathfrak{q}_j^1, \ldots, \mathfrak{q}_j^M, \mathfrak{q}_j^h \in \mathbb{R}_+^n \text{ and } \mathfrak{q}_j^1 + \ldots + \mathfrak{q}_j^M + \mathfrak{q}_j^h = \mathbf{q}_j.\end{aligned} \tag{2.6}$$

The interpretation is directly analogous to that for the two-member model. Using (2.6), we have the following definition.

Definition 2.11. *Let $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \ldots, T\}$ be a set of observations. A combination of M utility functions $U^1, \ldots, U^M$ provides an* M-member collective rationalization (CR-M) *of S if for each observation j there exist feasible personalized quantities $\widehat{\mathbf{q}}_j = (\mathfrak{q}_j^1, \ldots, \mathfrak{q}_j^M, \mathfrak{q}_j^h)$ and $\mu_j^2, \ldots, \mu_j^M \in \mathbb{R}_{++}$ such that*

$$U^1(\widehat{\mathbf{q}}_j) + \sum_{m=2}^{M} \mu_j^m U^2(\widehat{\mathbf{q}}_j) \geq U^1(\widehat{\mathbf{z}}) + \sum_{m=2}^{M} \mu_j^m U^2(\widehat{\mathbf{z}})$$

for all $\widehat{\mathbf{z}} = (\mathfrak{z}^1, \ldots, \mathfrak{z}^M, \mathfrak{z}^h)$ *with* $\mathfrak{z}^1, \ldots, \mathfrak{z}^M, \mathfrak{z}^h \in \mathbb{R}_+^n$ *and* $\mathbf{p}_j'(\mathfrak{z}^1 + \ldots + \mathfrak{z}^M + \mathfrak{z}^h) \leq \mathbf{p}_j'\mathbf{q}_j$.

Analogous to the two-member case, optimal household quantities result from the maximization of a weighted sum of household member utilities, with weights representing the bargaining power of the household members. Once more, optimality is to be understood in a Pareto efficiency sense.

To introduce the collective rationalization conditions for the M-member case, we define *feasible personalized prices* $(\widehat{\mathbf{p}}_j^1, \ldots, \widehat{\mathbf{p}}_j^M)$ as

$$\begin{aligned}
&\widehat{\mathbf{p}}_j^m = (\mathfrak{p}_j^{1,m}, \ldots, \mathfrak{p}_j^{M,m}, \mathfrak{p}_j^{h,m}) \text{ for } m = 1, \ldots, M-1 \text{ and} \\
&\widehat{\mathbf{p}}_j^M = (\mathbf{p}_j - \sum_{m=1}^{M-1} \mathfrak{p}_j^{1,m}, \ldots, \mathbf{p}_j - \sum_{m=1}^{M-1} \mathfrak{p}_j^{M,m}, \mathbf{p}_j - \sum_{m=1}^{M-1} \mathfrak{p}_j^{h,m}), \\
&\text{with} \\
&\mathfrak{p}_j^{1,m}, \mathfrak{p}_j^{2,m}, \mathfrak{p}_j^{h,m} \in \mathbb{R}_+^n \text{ and } \sum_{m=1}^{M-1} \mathfrak{p}_j^{c,m} \leq \mathbf{p}_j \ (c = 1, \ldots, M, h);
\end{aligned} \tag{2.7}$$

and a *set of feasible personalized prices and quantities*

$$\widehat{S} = \{(\widehat{\mathbf{p}}_j^1, \ldots, \widehat{\mathbf{p}}_j^M; \widehat{\mathbf{q}}_j); j = 1, \ldots, T\}.$$

Once more, the interpretation is analogous to that for the two-member case. We then have the following result, which generalizes Proposition 2.2.

Proposition 2.12. *Let* $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \ldots, T\}$ *be a set of observations. The following conditions are equivalent:*

(i) there exists a combination of M *concave and continuous utility functions* $U^1, \ldots, U^M$ *that provide a* CR-M *of* S;

(ii) there exists a set of feasible personalized prices and quantities $\widehat{S}$ *such that for each* $m = 1, \ldots, M$ *the sets* $\{(\widehat{\mathbf{p}}_j^m; \widehat{\mathbf{q}}_j);\ j = 1, \ldots, T\}$ *all satisfy* GARP;

(iii) there exists a set of feasible personalized prices and quantities $\widehat{S}$ *and numbers* U_j^m, $\lambda_j^m > 0$ *such that for all* $i, j \in \{1, \ldots, T\} : U_i^m - U_j^m \leq \lambda_j^m (\widehat{\mathbf{p}}_j^m)'(\widehat{\mathbf{q}}_i - \widehat{\mathbf{q}}_j)$ *with* $(m = 1, \ldots, M)$.

Note that this proposition naturally complies with the unitary *GARP* condition if $M = 1$.

2.5.2 Minimum number of goods and observations to enable rejection of M-member collective rationality

Let us then regard the minimal empirical conditions for possible rejection of the *CR-M* conditions in Proposition 2.12. These are given in the following result, which generalizes Proposition 2.8.

Proposition 2.13. *There does not always exist a combination of utility functions* $U^1, \ldots, U^M$ *that provide a* CR-M *of the set of observations* $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \ldots, T\}$ *if and only if (i) the number of goods* $n \geq M+1$ *and (ii) the number of observations* $T \geq M+1$.

In words, as soon as there are more goods and observations than household members, the collective model can be rejected. If one of the conditions (i) and (ii) in Proposition 2.13 is not fulfilled, then a *CR-M* of the set of observations S is always possible.

To further illustrate, we next provide a general price-quantity data structure that cannot be collectively rationalized.

Example 2.14. *In the proof of Proposition 2.13, we establish that a* CR-M *of the set* $S = \{(\mathbf{p}_j; \mathbf{q}_j), j = 1, \ldots, M+1\}$ *is impossible if the following conditions are met:*

$$\forall j \in \{1, \ldots, M+1\} : \mathbf{p}'_j \mathbf{q}_j > \mathbf{p}'_j \Big(\sum_{i=1, i \neq j}^{M+1} \mathbf{q}_i \Big). \tag{2.8}$$

We investigate these conditions for $\mathbf{p}_j \in \mathbb{R}^{M+1}_{++}$ *and* $\mathbf{q}_j \in \mathbb{R}^{M+1}_{+}$ $(j = 1, \ldots, M+1)$ *that have the following structure:*

$$\mathbf{p}_j = (\,1 \cdots 1\; p\; 1 \cdots 1\,)' \text{ and } \mathbf{q}_j = (\,1 \cdots 1\; q\; 1 \cdots 1\,)',$$

where p *and* q *always appear as the* j*-th row elements of respectively* $\mathbf{p}_j$ *and* $\mathbf{q}_j$*. This specific set of observations* S *obtains:*

$$\begin{aligned} \mathbf{p}'_j \mathbf{q}_j &= pq + M \quad \forall j \in \{1, \ldots, M+1\}, \text{ and} \\ \mathbf{p}'_j \mathbf{q}_i &= p + q + M - 1 \quad \forall i, j \in \{1, \ldots, M+1\}, j \neq i. \end{aligned}$$

Hence, the set S *meets 2.8 if and only if*

$$pq + M > M(p + q + M - 1). \tag{2.9}$$

Rewriting 2.9 as

$$p(q - M) > M(q + M - 2),$$

it is easy to see that for all $q > M$ *there exists* p *such that 2.9 is met.*

To give a numerical example, we reject collective rationality for $M = 5$ *if* $q = 10$ *and* $p = 14$. *Similar constructions are conceivable for alternative* M *values.*

2.5.3 Testable collective rationality restrictions

We next generalize the testable collective rationality restrictions of Section 2.3 and 2.4. First, as for the necessity restrictions, we can establish similar results as Lemmas 2.4 and 2.6. For compactness, we abstract from a formal statement, but the analogy with the two-member case is easy. Using this, we have the following result.

Proposition 2.15. *Suppose that there exists a combination of utility functions* $U^1, \ldots, U^M$ *that provide a* CR-M *of the set of observations* $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \ldots, T\}$. *Then there exist* hypothetical *relations* H_0^m, H^m *for each member* $m \in \{1, \ldots, M\}$ *such that:*

(i) if $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$ *then* $\mathbf{q}_i\ H_0^m\ \mathbf{q}_j$ *for some* m;

(ii) if $\mathbf{q}_i\ H_0^m\ \mathbf{q}_k$, $\mathbf{q}_k\ H_0^m\ \mathbf{q}_l$, $\ldots$, $\mathbf{q}_z\ H_0^m\ \mathbf{q}_j$ *for some (possibly empty) sequence* $(k, l, \ldots, z)$ *then* $\mathbf{q}_i\ H^m\ \mathbf{q}_j$;

(iii) for $M^* \leq M$ *and* $\mathbf{M} \subsetneq \{1, ..., M\}$: *if* $\mathbf{p}_s'\mathbf{q}_s \geq \mathbf{p}_s'(\sum_{k=1}^{M^*}\mathbf{q}_{t_k})$ *and for all* $m \in \mathbf{M}$ *we have* $\mathbf{q}_{t_{k(m)}} H^m \mathbf{q}_s$ *for some* $k(m) \leq M^*$, *then* $\mathbf{q}_s\ H_0^l\ \mathbf{q}_{t_k}$ *for some* $l \notin \mathbf{M}$ *and* $k \leq M^*$;

(iv) for $M^* \leq M$ *and* $\mathbf{M} \subsetneq \{1, ..., M\}$: *if* $\mathbf{p}_s'\mathbf{q}_s \geq \mathbf{p}_s'(\sum_{k=1}^{M^*}\mathbf{q}_{t_k})$ *and for all* $m \in \mathbf{M}$ *we have* $\mathbf{q}_{t_{k(m)}} H^m \mathbf{q}_s$ *for some* $k(m) \leq M^* - 1$, *then* $\mathbf{q}_s\ H_0^l\ \mathbf{q}_{t_{M^*}}$ *for some* $l \notin \mathbf{M}$;

(v) for $M^* \leq M$: *if for all* m *we have* $\mathbf{q}_{s_{k(m)}} H^m \mathbf{q}_t$ *for some* $k(m) \leq M^*$, *then* $\mathbf{p}_t'\mathbf{q}_t \leq \mathbf{p}_t'(\sum_{k=1}^{M^*}\mathbf{q}_{s_k})$.

This necessary condition has a directly similar interpretation as its two-member analogue. Rules (i)-(iv) contain restrictions on the specification of the hypothetical relations H_0^m, H^m for the given set of observations S. Rule (v), which complies with rules (v)-(vi) in Proposition 2.7, subsequently states that, if each household member m prefers $\mathbf{q}_{i_m}$ over $\mathbf{q}_j$, then $\mathbf{q}_j$ cannot be more expensive than the combination of these preferred quantities under the prices $\mathbf{p}_j$. It is easy to verify that this condition reduces to the unitary $GARP$ condition for $M = 1$ (i.e. there is only one household member).

Note that we do not need to extend rule (iv) towards right hand sides with more than two terms since these are already captured by rule (iv). To be more precise, if $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'(\mathbf{q}_{j_1} + \cdots + \mathbf{q}_{j_k})$ with $k > 2$ then we *a fortiori* also have that $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'(\mathbf{q}_{j_s} + \mathbf{q}_{j_t})$ for any $j_s, j_t \in \{j_1, \ldots j_k\}$. Therefore our rule (iv) refines the rule (iv) of Proposition S4 of Cherchye, De Rock and Vermeulen (2007s).
We next define the complementary sufficiency condition for a *CR-M* of the set of observations S.

Proposition 2.16. *Let $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \ldots, T\}$ be a set of observations. Suppose that there exist* hypothetical *relations H_0^m, H^m for each member $m \in \{1, \ldots, M\}$ that satisfy rules (i)-(v) in Proposition 2.15 and in addition allow for constructing sets $S^1, \ldots, S^M$ with $\cup_m S^m = S$ and $S^m \cap S^l = \varnothing$ for $m \neq l$ such that*
(v) $S^m = \{(\mathbf{p}_j; \mathbf{q}_j) \in S \mid \mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'\mathbf{q}_i$ whenever $\mathbf{q}_i\ H^m\ \mathbf{q}_j\}$;
(vi) for each $(\mathbf{p}_i; \mathbf{q}_i), (\mathbf{p}_j; \mathbf{q}_j) \in S^m :\ \mathbf{q}_i H_0^m \mathbf{q}_j$ whenever $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$.
Then there exists a combination of utility functions $U^1, \ldots, U^M$ that provide a CR-M *of the set S.*

Like in the two-member case, this sufficient condition can be interpreted in terms of a *situation-dependent dictatorship* model. Just like the necessity condition, the sufficiency condition reduces to the *GARP* condition for $M = 1$. In that case, the only feasible personalized prices and quantities are the observed aggregate prices and quantities, and the necessary and sufficient conditions for rational household behavior always coincide. In the more general case ($M > 1$), we may expect the necessity condition to converge towards the sufficiency condition when the sample size increases; compare with our discussion in Section 2.4.

Finally, since the necessary and sufficient conditions in Propositions 2.15 and 2.16 only require aggregate prices $\mathbf{p}_j$and quantities $\mathbf{q}_j$, they enable operational collective rationality tests that apply to the general case of T observations. Finite algorithms for verifying the conditions are directly similar to the one present in Chapter 3 for the two-member case.

2.6 Summary and concluding remarks

To conclude, we recall that the collective model under study considers general member-specific preferences, and only assumes that the empirical analyst observes the aggregate household consumption quantities and prices. Attractively, the model encompasses a large variety of alternative behavioral models as special cases, which include additional prior information that implies extra restrictions regarding the feasible personalized quantities and prices (see (2.6) and (2.7) for the general model under study). For example, such additional structure may pertain to observability of private and/or public consumption quantities or to the nature of the individual members' preferences (namely, egoistic rather than altruistic); notable cases are the traditional unitary model and the collective model of Chiappori (1988). As we will show in the following chapters, such special cases entail in general more stringent testable necessary and sufficient conditions for collective rationalization that solely use observed prices and quantities.

As a final note, we recall that the testable collective rationality conditions in Propositions 2.15 and 2.16 have an analogous structure as the (unitary) *GARP*, which allows for easy adaptations of the existing power and goodness-of-fit measures for nonparametric consumption analysis (see respectively Bronars, 1987, and Varian, 1990, for inspiring results and our applications in the Chapters 3 and 4 for examples). Specifically, using the necessary and sufficient conditions one can generate upper and lower bounds for each of these measures. [If these upper and lower bounds are situated close to each other, one possible interpretation is that the empirical content of the necessary and sufficient conditions is practically the same for the set of observations under study.]

Appendix

Proof of Proposition 2.2:

Varian (1982) proves the equivalence between conditions (ii) and (iii) of the proposition. Therefore, it suffices to prove equivalence between (i) and (iii).[18]

(*(i)* $\Rightarrow$ *(iii)*) Under condition (i), for each observation j there exists $\widehat{\mathbf{q}}_j = (\mathfrak{q}_j^1, \mathfrak{q}_j^2, \mathfrak{q}_j^h)$ that solves the problem (for $\widehat{\mathbf{z}} = (\mathfrak{z}^1, \mathfrak{z}^2, \mathfrak{z}^h)$ with $\mathfrak{z}^1, \mathfrak{z}^2, \mathfrak{z}^h \in \mathbb{R}_+^n$)

$$\max_{\widehat{\mathbf{z}}} U^1(\mathfrak{z}^1, \mathfrak{z}^2, \mathfrak{z}^h) + \mu_j U^2(\mathfrak{z}^1, \mathfrak{z}^2, \mathfrak{z}^h) \text{ s.t. } \mathbf{p}_j'(\mathfrak{z}^1 + \mathfrak{z}^2 + \mathfrak{z}^h) \leq \mathbf{p}_j'\mathbf{q}_j.$$

Given concavity, both individual utility functions are subdifferentiable, which carries over to their weighted sum $U^1 + \mu_j U^2$.[19] An optimal solution to the above maximization problem must therefore satisfy (for η_j the Lagrange multiplier associated with the budget constraint)

$$U^1_{\mathfrak{q}_j^c} + \mu_j U^2_{\mathfrak{q}_j^c} \leq \eta_j \mathbf{p}_j,$$

where $U^m_{\mathfrak{q}_j^c}$ $(m = 1, 2)$ is a subgradient of the utility function U^m defined for the vector $\mathfrak{z}^c \in \mathbb{R}_+^n$ and evaluated at $\mathfrak{q}_j^c$ $(c = 1, 2, h)$. Letting $\mathfrak{p}_j^c = \frac{U^1_{\mathfrak{q}_j^c}}{\eta_j}$, $\lambda_j^1 = \eta_j$ and $\lambda_j^2 = \frac{\eta_j}{\mu_j}$ thus gives

$$U^1_{\mathfrak{q}_j^c} = \lambda_j^1 \mathfrak{p}_j^c \text{ and } U^2_{\mathfrak{q}_j^c} \leq \lambda_j^2(\mathbf{p}_j - \mathfrak{p}_j^c). \tag{2.10}$$

Next, concavity of the functions U^1 and U^2 implies $(m = 1, 2)$

$$U^m(\widehat{\mathbf{q}}_i) - U^m(\widehat{\mathbf{q}}_j) \leq \sum_{c=1,2,h} U^m_{\mathfrak{q}_j^c}(\mathfrak{q}_i^c - \mathfrak{q}_j^c). \tag{2.11}$$

Substituting (2.10) in (2.11) and setting $U_k^m = U^m(\widehat{\mathbf{q}}_k)$ $(m = 1, 2;$ $k = i, j)$ obtains the conditions (iii) of the proposition.

[18] This proof generalizes that of Chiappori (1988), who focuses on the specific case of household labor supply. Another difference is that Chiappori focuses on (a strong version of) the *SARP* conditions (see Chiappori and Rochet, 1987, p. 688, for a precise definition) while our proof uses the (less stringent) *GARP* conditions. It is worth pointing out that all our results for the *GARP* can be adapted to apply for the (strong) *SARP*.

[19] To be precise, $-U^m$ $(m = 1, 2)$ is convex and therefore subdifferentiable. This, of course, does not affect our argument.

(*(iii)* $\Rightarrow$ *(i)*) Under condition (iii), we can define for any $\widehat{\mathbf{q}}$=$(\mathfrak{q}^1, \mathfrak{q}^2, \mathfrak{q}^h)$ such that $\mathbf{p}_j'(\mathfrak{q}^1 + \mathfrak{q}^2 + \mathfrak{q}^h) \leq \mathbf{p}_j'\mathbf{q}_j$

$$U^1(\widehat{\mathbf{q}}) = \min_{i\in\{1,\ldots,T\}} \left[U_i^1 + \lambda_i^1(\widehat{\mathbf{p}}_i^1)'(\widehat{\mathbf{q}} - \widehat{\mathbf{q}}_i)\right] \text{ and} \tag{2.12}$$

$$U^2(\widehat{\mathbf{q}}) = \min_{i\in\{1,\ldots,T\}} \left[U_i^2 + \lambda_i^2(\widehat{\mathbf{p}}_i^2)'(\widehat{\mathbf{q}} - \widehat{\mathbf{q}}_i)\right]. \tag{2.13}$$

Varian (1982) proves that $U^1(\widehat{\mathbf{q}}_j) = U_j^1$ and $U^2(\widehat{\mathbf{q}}_j) = U_j^2$. Next, given $\mu_j \in \mathbb{R}_{++}$, we have that

$$U^1(\widehat{\mathbf{q}}) + \mu_j U^2(\widehat{\mathbf{q}}) \leq U_j^1 + \lambda_j^1(\widehat{\mathbf{p}}_j^1)'(\widehat{\mathbf{q}} - \widehat{\mathbf{q}}_j) + \mu_j\left[U_j^2 + \lambda_j^2(\widehat{\mathbf{p}}_j^2)'(\widehat{\mathbf{q}} - \widehat{\mathbf{q}}_j)\right].$$

Without losing generality, we concentrate on $\mu_j = (\lambda_j^1/\lambda_j^2)$, which obtains

$$U^1(\widehat{\mathbf{q}}) + \mu_j U^2(\widehat{\mathbf{q}}) \leq U_j^1 + \mu_j U_j^2 + \lambda_j^1(\mathbf{p}_j)'(\mathbf{q} - \mathbf{q}_j),$$

where $\mathbf{q} = (\mathfrak{q}^1 + \mathfrak{q}^2 + \mathfrak{q}^h)$. Since $\mathbf{p}_j'\mathbf{q} \leq \mathbf{p}_j'\mathbf{q}_j$, we thus have

$$U^1(\widehat{\mathbf{q}}) + \mu_j U^2(\widehat{\mathbf{q}}) \leq U_j^1 + \mu_j U_j^2 = U^1(\widehat{\mathbf{q}}_j) + \mu_j U^2(\widehat{\mathbf{q}}_j),$$

which proves that $\widehat{\mathbf{q}}_j$ maximizes $U^1(\widehat{\mathbf{q}}) + \mu_j U^2(\widehat{\mathbf{q}})$ subject to the constraint $\mathbf{p}_j'(\mathfrak{q}^1 + \mathfrak{q}^2 + \mathfrak{q}^h) \leq \mathbf{p}_j'\mathbf{q}_j$. We conclude that the functions U^1 and U^2 in (2.12)-(2.13) provide a collective rationalization of S. These functions satisfy the conditions in part (i) of the proposition (compare with Varian, 1982). *Q.E.D.*

Proof of Lemma 2.4:

(Necessity) We first derive that $\mathbf{q}_i\ R_0\ \mathbf{q}_j$ implies $\widehat{\mathbf{q}}_i\ R_0^1\ \widehat{\mathbf{q}}_j$ or $\widehat{\mathbf{q}}_i\ R_0^2\ \widehat{\mathbf{q}}_j$ for any set $\widehat{S}$. The result follows from the fact that $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$ (or $\mathbf{q}_i\ R_0\ \mathbf{q}_j$) is incompatible with the existence of some $\widehat{S}$ such that $(\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_i < (\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_j$ and $(\widehat{\mathbf{p}}_i^2)'\widehat{\mathbf{q}}_i < (\widehat{\mathbf{p}}_i^2)'\widehat{\mathbf{q}}_j$. Indeed, summing these last inequalities immediately yields $\mathbf{p}_i'\mathbf{q}_i < \mathbf{p}_i'\mathbf{q}_j$.

(Sufficiency) We next derive that if, for all sets of feasible personalized prices and quantities $\widehat{S}$, $\widehat{\mathbf{q}}_i\ R_0^1\ \widehat{\mathbf{q}}_j$ or $\widehat{\mathbf{q}}_i\ R_0^2\ \widehat{\mathbf{q}}_j$ then $\mathbf{q}_i\ R_0\ \mathbf{q}_j$. The result is obtained by noting that $\mathbf{p}_i'\mathbf{q}_i < \mathbf{p}_i'\mathbf{q}_j$ implies $(\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_i + (\widehat{\mathbf{p}}_i^2)'\widehat{\mathbf{q}}_i < (\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_j + (\widehat{\mathbf{p}}_i^2)'\widehat{\mathbf{q}}_j$ for all $\widehat{S}$. It is then easy to see that, if $\mathbf{p}_i'\mathbf{q}_i < \mathbf{p}_i'\mathbf{q}_j$, then there exists $\widehat{S}$ such that $(\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_i < (\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_j$ *and* $(\widehat{\mathbf{p}}_i^2)'\widehat{\mathbf{q}}_i < (\widehat{\mathbf{p}}_i^2)'\widehat{\mathbf{q}}_j$ (i.e. we have *neither* $\widehat{\mathbf{q}}_i\ R_0^1\ \widehat{\mathbf{q}}_j$ *nor* $\widehat{\mathbf{q}}_i\ R_0^2\ \widehat{\mathbf{q}}_j$); e.g. one may use $\mathfrak{p}_k^1 = (1/2)\mathbf{p}_k$ and $\mathfrak{q}_k^1 = \mathbf{q}_k$ $(k = i, j)$. Hence, we have for all sets $\widehat{S}$ that $\widehat{\mathbf{q}}_i\ R_0^1\ \widehat{\mathbf{q}}_j$ or $\widehat{\mathbf{q}}_i\ R_0^2\ \widehat{\mathbf{q}}_j$ only if $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$, i.e. $\mathbf{q}_i\ R_0\ \mathbf{q}_j$. *Q.E.D.*

Proof of Lemma 2.6:

Given that a collective rationalization of the set of observations S is possible, we consider a set $\widehat{S}$ that is consistent with condition (ii) in Proposition 2.2. Using Definition 2.3, this set $\widehat{S}$ defines relations R_0^m and R^m $(m = 1, 2)$. We will show that these relations satisfy rules (i)-(iv) in Lemma 2.6.

As for rule (i), we establish that, if $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$ and $\widehat{\mathbf{q}}_j\ R^1\ \widehat{\mathbf{q}}_i$, then $\widehat{\mathbf{q}}_i\ R_0^2\ \widehat{\mathbf{q}}_j$ (the argument for the other case is directly analogous). For $\widehat{\mathbf{q}}_j\ R^1\ \widehat{\mathbf{q}}_i$, consistency with condition (ii) in Proposition 2.2 requires $(\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_i \leq (\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_j$. Given $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$, this last inequality implies $(\widehat{\mathbf{p}}_i^2)'\widehat{\mathbf{q}}_i \geq (\widehat{\mathbf{p}}_i^2)'\widehat{\mathbf{q}}_j$ or $\widehat{\mathbf{q}}_i\ R_0^2\ \widehat{\mathbf{q}}_j$, which gives the result.

To derive rule (ii), suppose that $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'(\mathbf{q}_{j_1} + \mathbf{q}_{j_2})$ in combination with $\widehat{\mathbf{q}}_{j_1}\ R^1\ \widehat{\mathbf{q}}_i$ while not $\widehat{\mathbf{q}}_i\ R_0^2\ \widehat{\mathbf{q}}_{j_2}$. On the one hand, not $\widehat{\mathbf{q}}_i\ R_0^2\ \widehat{\mathbf{q}}_{j_2}$ means that $(\widehat{\mathbf{p}}_i^2)'\widehat{\mathbf{q}}_i < (\widehat{\mathbf{p}}_i^2)'\widehat{\mathbf{q}}_{j_2}$. On the other hand, $\widehat{\mathbf{q}}_{j_1}\ R^1\ \widehat{\mathbf{q}}_i$ requires that $(\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_i \leq (\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_{j_1}$ for the consistency with condition (ii) in Proposition 2.2. Combining these two inequalities would imply $\mathbf{p}_i'\mathbf{q}_i < (\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_{j_1} + (\widehat{\mathbf{p}}_i^2)'\widehat{\mathbf{q}}_{j_2} \leq \mathbf{p}_i'(\mathbf{q}_{j_1} + \mathbf{q}_{j_2})$, which contradicts $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'(\mathbf{q}_{j_1} + \mathbf{q}_{j_2})$. Thus, we conclude that $(\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'(\mathbf{q}_{j_1} + \mathbf{q}_{j_2}) \wedge \widehat{\mathbf{q}}_{j_1}\ R^1\ \widehat{\mathbf{q}}_i) \Rightarrow \widehat{\mathbf{q}}_i\ R_0^2\ \widehat{\mathbf{q}}_{j_2}$. A directly analogous argument holds for the other case.

As for rules (iii) and (iv), under $\widehat{\mathbf{q}}_{i_1}\ R^1\ \widehat{\mathbf{q}}_j$ and $\widehat{\mathbf{q}}_{i_2}\ R^2\ \widehat{\mathbf{q}}_j$ consistency with condition (ii) in Proposition 2.2 is obtained only if $(\widehat{\mathbf{p}}_j^1)'\widehat{\mathbf{q}}_j \leq (\widehat{\mathbf{p}}_j^1)'\widehat{\mathbf{q}}_{i_1}$ and $(\widehat{\mathbf{p}}_j^2)'\widehat{\mathbf{q}}_j \leq (\widehat{\mathbf{p}}_j^2)'\widehat{\mathbf{q}}_{i_2}$. This last result immediately yields $\mathbf{p}_j'\mathbf{q}_j \leq (\widehat{\mathbf{p}}_j^1)'\widehat{\mathbf{q}}_{i_1} + (\widehat{\mathbf{p}}_j^2)'\widehat{\mathbf{q}}_{i_2} \leq \mathbf{p}_j'(\mathbf{q}_{i_1} + \mathbf{q}_{i_2})$ if $\mathbf{q}_{i_1} \neq \mathbf{q}_{i_2}$ and, similarly, $\mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'\mathbf{q}_i$ if $\mathbf{q}_{i_1} = \mathbf{q}_{i_2} = \mathbf{q}_i$. *Q.E.D.*

Proof of Proposition 2.7:

The result follows immediately from combining Lemmas 2.4 and 2.6, when replacing the relations R_0^m and R^m by their hypothetical counterparts H_0^m and H^m. Rule (i) follows from Lemma 2.4. Rule (ii) defines the transitive closures H^1 and H^2 of the relations H_0^1 and H_0^2; compare with Definition 2.3. Finally, rules (iii)-(vi) follow from rules (i)-(iv) in Lemma 2.6. *Q.E.D.*

Proof of the result in Example 2.9:

For the specific data structure, consistency with the condition in Proposition 2.7 implies that there exist hypothetical relations that must satisfy for all $i, j \in \{1, \ldots, 7\}$, $i \neq j$: $\mathbf{q}_i\ H^m\ \mathbf{q}_j$ and not $\mathbf{q}_i\ H^l\ \mathbf{q}_j$ for

$m \neq l$; and we cannot have $\mathbf{q}_i\ H^1\ \mathbf{q}_k$ and $\mathbf{q}_j\ H^2\ \mathbf{q}_k$ for $k \in \{1, 7\}$ and for all $i, j \in \{1, \ldots, 7\} \backslash \{k\}$. Given this, one possible specification of the relations H_0^m, H^m is[20]

$$\forall i, j \in \{1, \ldots, 7\} : (i > j \Rightarrow \mathbf{q}_j\ H^1\ \mathbf{q}_i) \text{ and } (i < j \Rightarrow \mathbf{q}_j\ H^2\ \mathbf{q}_i).$$

Combining the corresponding requirements following from condition (ii) in Proposition 2.2 obtains for all $i \in \{2, \ldots, 6\}$ and $j \in \{1, \ldots, 7\}$

$$\begin{aligned} &(i > j \Rightarrow \mathbf{p}_i'\mathbf{q}_j - \varepsilon \leq (\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_j \leq \mathbf{p}_i'\mathbf{q}_j) \\ &\quad \text{and } (i < j \Rightarrow 0 \leq (\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_j \leq \varepsilon). \end{aligned} \tag{2.14}$$

Next, because $(\mathbf{q}_j)_e = (\mathfrak{q}_j^1)_e + (\mathfrak{q}_j^2)_e + (\mathfrak{q}_j^h)_e$ and $\mathfrak{p}_i^c \leq \mathbf{p}_i$ $(c = 1, 2, h)$, we obtain that $\mathbf{p}_i'\mathbf{q}_j - \varepsilon \leq (\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_j \leq \mathbf{p}_i'\mathbf{q}_j$ implies for all $e \in \{1, \ldots, n\}$

$$(\mathbf{p}_i)_e(\mathbf{q}_j)_e - \varepsilon \leq \sum_{c \in \{1,2,h\}} (\mathfrak{p}_i^c)_e(\mathfrak{q}_j^c)_e \leq (\mathbf{p}_i)_e(\mathbf{q}_j)_e,$$

which in turn entails for all $c \in \{1, 2, h\}$ with $(\mathfrak{q}_j^c)_e > 0$

$$(\mathbf{p}_i)_e - \frac{\varepsilon}{(\mathfrak{q}_j^c)_e} \leq (\mathfrak{p}_i^c)_e \leq (\mathbf{p}_i)_e.$$

Similarly, the restriction $0 \leq (\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_j \leq \varepsilon$ requires

$$\left[0 \leq \sum_{c \in \{1,2,h\}} (\mathfrak{p}_i^c)_e(\mathfrak{q}_j^c)_e \leq \varepsilon\right] \Rightarrow \left[\forall c \in \{1, 2, h\} : 0 \leq (\mathfrak{p}_i^c)_e \leq \frac{\varepsilon}{(\mathfrak{q}_j^c)_e}\right].$$

Let us take $e = 1$ and consider $0 < \sigma = \min\limits_{j \in \{1,\ldots,7\}, e \in \{1,\ldots,n\}} (\mathbf{q}_j)_e$. The Pigeon Hole Principle implies $\forall j \in \{1, \ldots, 7\} : \exists c_j \in \{1, 2, h\} : (\mathfrak{q}_j^{c_j})_1 \geq (\sigma/3)$, so that we get

$$\begin{aligned} &\left[\mathbf{p}_i'\mathbf{q}_j - \varepsilon \leq (\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_j \leq \mathbf{p}_i'\mathbf{q}_j\right] \Rightarrow \\ &\left[\exists c_j \in \{1, 2, h\} : (\mathbf{p}_i)_1 - \frac{3\varepsilon}{\sigma} \leq (\mathfrak{p}_i^{c_j})_1 \leq (\mathbf{p}_i)_1\right] \end{aligned}$$

and

[20] The following argument can be repeated for any alternative specification of the relations H_0^m, H^m that meets the necessity condition in Proposition 2.7.

$$\left[0 \leq (\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_j \leq \varepsilon\right] \Rightarrow \left[\exists c_j \in \{1,2,h\} : 0 \leq (\mathfrak{p}_i^{c_j})_1 \leq \frac{3\varepsilon}{\sigma}\right].$$

Remark that $\frac{\min\limits_{j,e}(\mathbf{p}_j)_e \min\limits_{j,e}(\mathbf{q}_j)_e}{6} > \varepsilon$ implies $(\mathbf{p}_i)_1 - \frac{3\varepsilon}{\sigma} > \frac{3\varepsilon}{\sigma}$. Using this, the preference structure in (2.14) obtains $\forall i \in \{2,\ldots,6\}$

$$\forall j_1, j_2 \in \{1,\ldots,7\} : (i > j_1 \wedge i < j_2 \Rightarrow c_{j_1} \neq c_{j_2}); \tag{2.15}$$

the reasoning is that $(i > j_1 \Rightarrow (\mathbf{p}_i)_1 - \frac{3\varepsilon}{\sigma} \leq (\mathfrak{p}_i^{c_{j_1}})_1 \leq (\mathbf{p}_i)_1)$ and $(i < j_2 \Rightarrow 0 \leq (\mathfrak{p}_i^{c_{j_2}})_1 \leq \frac{3\varepsilon}{\sigma})$, which excludes $c_{j_1} = c_{j_2}$. Inconsistency with the collective rationalization conditions in Proposition 2.2 follows as (2.15) implies $c_{j_1} \neq c_{j_2}$ for all $j_1, j_2 \in \{1,3,5,7\}, j_1 \neq j_2$; and this contradicts $c_j \in \{1,2,h\}$ $\forall j \in \{1,\ldots,7\}$. *Q.E.D.*

Proof of Proposition 2.10:

Suppose that we can construct sets S^1 and S^2 in Proposition 2.10. Then we can construct a set of feasible prices and quantities $\widehat{S}$ that meets condition (ii) in Proposition 2.2. Specifically, define $\widehat{S}$ such that

if $(\mathbf{p}_j; \mathbf{q}_j) \in S^1$ then $\mathfrak{q}_j^1 = \mathbf{q}_j$ (and thus $\mathfrak{q}_j^2 = \mathfrak{q}_j^h = \mathbf{0}$);
if $(\mathbf{p}_j; \mathbf{q}_j) \in S^2$ then $\mathfrak{q}_j^2 = \mathbf{q}_j$ (and thus $\mathfrak{q}_j^1 = \mathfrak{q}_j^h = \mathbf{0}$);
and $\mathfrak{p}_j^1 = \mathbf{p}_j$, $\mathfrak{p}_j^2 = \mathfrak{p}_j^h = \mathbf{0}$ for all $(\mathbf{p}_j; \mathbf{q}_j) \in S$.

We restrict attention to household member 1; but a directly analogous reasoning applies to member 2. Condition (ii) in Proposition 2.2 states that $(\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_i \geq (\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_k, \ldots, (\widehat{\mathbf{p}}_z^1)'\widehat{\mathbf{q}}_z \geq (\widehat{\mathbf{p}}_z^1)'\widehat{\mathbf{q}}_j$ for some (possibly empty) sequence $(k, \ldots, z)$ implies $(\widehat{\mathbf{p}}_j^1)'\widehat{\mathbf{q}}_j \leq (\widehat{\mathbf{p}}_j^1)'\widehat{\mathbf{q}}_i$. As a preliminary step, we note that under the above specification of the set $\widehat{S}$ we have for all $(\mathbf{p}_{l_1}; \mathbf{q}_{l_1}) \in S^1$ that $(\widehat{\mathbf{p}}_{l_1}^1)'\widehat{\mathbf{q}}_{l_2} = 0$ if $(\mathbf{p}_{l_2}; \mathbf{q}_{l_2}) \in S^2$. This makes that the only interesting case is $(\mathbf{p}_l; \mathbf{q}_l) \in S^1$ for all $l = i, j, k, \ldots, z$. Hence, obtaining $(\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_i \geq (\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_k, \ldots, (\widehat{\mathbf{p}}_z^1)'\widehat{\mathbf{q}}_z \geq (\widehat{\mathbf{p}}_z^1)'\widehat{\mathbf{q}}_j \Rightarrow (\widehat{\mathbf{p}}_j^1)'\widehat{\mathbf{q}}_j \leq (\widehat{\mathbf{p}}_j^1)'\widehat{\mathbf{q}}_i$ boils down to verifying $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_k, \ldots, \mathbf{p}_z'\mathbf{q}_z \geq \mathbf{p}_z'\mathbf{q}_j \Rightarrow \mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'\mathbf{q}_i$ for any possible sequence of $(i, k, \ldots, z, j)$ with $(\mathbf{p}_l; \mathbf{q}_l) \in S^1$ for all $l = i, j, k, \ldots, z$.

Using rule (viii) in Proposition 2.10, we have $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_k, \ldots, \mathbf{p}_z'\mathbf{q}_z \geq \mathbf{p}_z'\mathbf{q}_j \Rightarrow \mathbf{q}_i\ H_0^1\ \mathbf{q}_k, \ldots, \mathbf{q}_z\ H_0^1\ \mathbf{q}_j$, which in turn implies $\mathbf{q}_i\ H^1\ \mathbf{q}_j$. Rule (vii) in Proposition 2.10 consequently guarantees $\mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'\mathbf{q}_i$, i.e. condition (ii) in Proposition 2.2 is met for member 1. *Q.E.D.*

Proof of Proposition 2.12:
The construction of the proof is directly analogous to that of Proposition 2.2.

Proof of Proposition 2.13:
(Necessity) The basic intuition is the same as the one explained in Section 2.4

(Sufficiency) Let $S = \{(\mathbf{p}_j; \mathbf{q}_j), j = 1, \dots, M+1\}$, then we show that a *CR-M* is impossible if the following conditions are met:

$$\forall j \in \{1, \dots, M+1\} : \mathbf{p}_j' \mathbf{q}_j > \mathbf{p}_j' (\sum_{i=1, i \neq j}^{M+1} \mathbf{q}_i). \tag{2.16}$$

As a preliminary step, we note that for all sets $\widehat{S}$ we have $\forall\ i, j \in \{1, .., M+1\}$:

$$\sum_{m=1}^{M} (\widehat{\mathbf{p}}_j^m)' \widehat{\mathbf{q}}_i = \mathbf{p}_j' \mathbf{q}_i; \tag{2.17}$$

this follows from the definitions of $\widehat{\mathbf{p}}_j^m$ and $\widehat{\mathbf{q}}_i$, and will prove useful in our following discussion.

Let us then rewrite the *CR-M* conditions (iii) of Proposition 2.12 as (for each $i, j \in \{1, .., M+1\}$ and $m \in \{1, \dots, M\}$)

$$\frac{1}{\lambda_j^m}(U_i^m - U_j^m) \leq (\widehat{\mathbf{p}}_j^m)'(\widehat{\mathbf{q}}_i - \widehat{\mathbf{q}}_j). \tag{2.18}$$

Next observe that, if there are $M+1$ observations, and given that there are only M household members, then for any possible ordering of each member m's ($m = 1, \dots, M$) 'utilities' U_k^m ($k = 1, \dots, M+1$) there is at least one observation $j \in \{1, \dots, M+1\}$ of which each m-th ($m = 1, \dots, M$) household member is dominated 'in utility terms' by some other observation $i(m) \in \{1, \dots, M+1\}$; i.e. $\exists j \in \{1, \dots, M+1\}$: $\forall m \in \{1, \dots, M\} : \exists i(m) \in \{1, \dots, M+1\}, i(m) \neq j : U_j^m \leq U_{i(m)}^m$.

Let us then concentrate on such an observation j when constructing necessary conditions for a *CR-M* of the set S. For all $m = 1, \dots, M$ it holds that (see (2.18))

$$0 \leq \frac{1}{\lambda_j^m}(U_{i(m)}^m - U_j^m) \leq (\widehat{\mathbf{p}}_j^m)'(\widehat{\mathbf{q}}_{i(m)} - \widehat{\mathbf{q}}_j),$$

or,

$$(\widehat{\mathbf{p}}_j^m)'\widehat{\mathbf{q}}_j \leq (\widehat{\mathbf{p}}_j^m)'\widehat{\mathbf{q}}_{i(m)}.$$

Using (2.17), we thus have

$$\mathbf{p}_j'\mathbf{q}_j = \sum_{m=1}^{M}(\widehat{\mathbf{p}}_j^m)'\widehat{\mathbf{q}}_j \leq \sum_{m=1}^{M}(\widehat{\mathbf{p}}_j^m)'\widehat{\mathbf{q}}_{i(m)}, \tag{2.19}$$

which provides a lower bound for $\sum\limits_{m=1,\ldots,M}(\widehat{\mathbf{p}}_j^m)'\widehat{\mathbf{q}}_{i(m)}$.

On the other hand, an upper bound can be constructed on the basis of (2.17), which implies for any subset $\mathbf{M} \subseteq \{1,\ldots,M\}$

$$\sum_{l\in\mathbf{M}}(\widehat{\mathbf{p}}_j^l)'\widehat{\mathbf{q}}_{i(m)} \leq \sum_{l=1}^{M}(\widehat{\mathbf{p}}_j^l)'\widehat{\mathbf{q}}_{i(m)} \leq \mathbf{p}_j'\mathbf{q}_{i(m)};\ \forall i(m), m \in \{1,\ldots,M\}.$$

Define $\mathbf{M}_i = \{m \in \{1,\ldots,M\} \mid i(m) = i\}$ for all $i \in \{1,\ldots,M+1\}$; note that $\mathbf{M}_j = \emptyset$ by construction. Then

$$\sum_{m=1}^{M}(\widehat{\mathbf{p}}_j^m)'\widehat{\mathbf{q}}_{i(m)} = \sum_{l\in\mathbf{M}_1}(\widehat{\mathbf{p}}_j^l)'\widehat{\mathbf{q}}_1 + \cdots + \sum_{l\in\mathbf{M}_{M+1}}(\widehat{\mathbf{p}}_j^l)'\widehat{\mathbf{q}}_{M+1} \leq \mathbf{p}_j'(\sum_{i=1,i\neq j}^{M+1}\mathbf{q}_i). \tag{2.20}$$

From (2.19) and (2.20), we derive a necessary condition for a *CR-M* of the set S

$$\mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'(\sum_{i=1,i\neq j}^{M+1}\mathbf{q}_i),$$

which conflicts with the property (2.16) of the observed prices and quantities under consideration.

We conclude that it is impossible to construct $U^1,\ldots,U^M$ that provide a *CR-M* of a set S that satisfies (2.16). This shows sufficiency for (at least) $M+1$ observations. Sufficiency for (at least) $M+1$ goods follows from Example 1 (CR-M rejection). *Q.E.D.*

Proof of Proposition 2.15:

The construction of the proof is directly analogous to that of Proposition 2.7.

Proof of Proposition 2.16:
It can be shown that, if the set of observations S meets the sufficiency condition, then the conditions for a *CR-M* of the data are met for the following specification of the set of feasible personalized prices and quantities $\widehat{S}$: (i) we specify the feasible personalized quantities such that for $(\mathbf{p}_i; \mathbf{q}_i) \in S^m : \mathfrak{q}_i^m = \mathbf{q}_i$ and $\mathfrak{q}_i^l = \mathbf{0}$ for $l \neq m$; (ii) for each observation i we specify feasible personalized prices such that $\mathfrak{p}_i^{m,m} = \mathbf{p}_i$ for $m \in \{1, \ldots, M-1\}$ and $\mathfrak{p}_i^{l,m} = \mathbf{0}$ for $l \neq m$. The construction of the proof is directly analogous to that of Proposition 2.10. *Q.E.D.*

Bargaining power and 'situation-dependent dictatorship'

This section shows that the requirement of a strictly positive bargaining power for each household member (i.e. $\infty > \mu_j > 0$ in Definition 2.1 of the main text) is compatible with the 'situation-dependent dictatorship' solution underlying the sufficiency result in Proposition 2.10. The following argument concentrates on two-member households for simplicity; but it can directly be generalized to hold for M-member households.

To formally explain the compatibility, we first recall the sufficiency part ("(iii) $\Rightarrow$ (i)") in the proof of Proposition 2.2. That proof shows that, if the data are consistent with the inequalities in part (iii) of Proposition 2.2, then consistency with a collective rationalization of the observed set S is possible for $\mu_j = (\lambda_j^1 / \lambda_j^2)$.

The proof of Proposition 2.10 then shows consistency with the *GARP* conditions in part (ii) of Proposition 2.2 for the following specification of the feasible personalized quantities and prices:

$$\begin{aligned}&\text{if } (\mathbf{p}_j; \mathbf{q}_j) \in S^1 \text{ then } \mathfrak{q}_j^1 = \mathbf{q}_j, \text{ and if } (\mathbf{p}_j; \mathbf{q}_j) \in S^2 \text{ then } \mathfrak{q}_j^2 = \mathbf{q}_j;\\ &\text{and } \mathfrak{p}_j^1 = \mathbf{p}_j,\ \mathfrak{p}_j^2 = \mathfrak{p}_j^h = \mathbf{0} \text{ for all } (\mathbf{p}_j; \mathbf{q}_j) \in S.\end{aligned}$$

We note that consistency with these *GARP* conditions implies consistency with the inequalities in part (iii) of Proposition 2.2. In fact, it can be verified that for the given specification of the feasible personalized quantities and prices, consistency with these inequalities does not in any way impose $\lambda_j^1 = 0$ or $\lambda_j^2 = 0$. For example, let us give a specific solution for the inequalities for member 1:

$$\forall(\mathbf{p}_i;\mathbf{q}_i) \in S^1, (\mathbf{p}_j;\mathbf{q}_j) \in S^1 : U_i^1 - U_j^1 \leq \lambda_j^1(\mathbf{p}_j)'(\mathbf{q}_i - \mathbf{q}_j); \quad (2.21)$$
$$\forall(\mathbf{p}_i;\mathbf{q}_i) \in S^1, (\mathbf{p}_j;\mathbf{q}_j) \in S^2 : U_i^1 - U_j^1 \leq \lambda_j^1(\mathbf{p}_j)'(\mathbf{q}_i); \quad (2.22)$$
$$\forall(\mathbf{p}_i;\mathbf{q}_i) \in S^2, (\mathbf{p}_j;\mathbf{q}_j) \in S^1 : U_i^1 - U_j^1 \leq -\lambda_j^1(\mathbf{p}_j)'(\mathbf{q}_j); \quad (2.23)$$
$$\forall(\mathbf{p}_i;\mathbf{q}_i) \in S^2, (\mathbf{p}_j;\mathbf{q}_j) \in S^2 : U_i^1 - U_j^1 \leq 0. \quad (2.24)$$

To construct the solution, we first take, for all $(\mathbf{p}_i;\mathbf{q}_i) \in S^1$, $U_i^1 > 0$ and $\lambda_i^1 > 0$ that obtain consistency with (2.21); this is possible because the subset S^1 is consistent with *GARP*. Subsequently, we specify for all $(\mathbf{p}_j;\mathbf{q}_j) \in S^2$ that $U_j^1 = \overline{U}$ with $\min\limits_{(\mathbf{p}_i;\mathbf{q}_i)\in S^1} U_i^1 > \overline{U}$ (> 0) (note that this guarantees consistency with (2.24)). In that case, we can always choose $\lambda_j^1 > 0$ for all $(\mathbf{p}_j;\mathbf{q}_j) \in S$; see in particular (2.22)-(2.23). We conclude that the set of inequalities (2.21)-(2.24) does not impose $\lambda_j^1 = 0$ for any j. A directly analogous argument obtains that we can always set $\lambda_j^2 > 0$ for any j when the sufficiency condition in Proposition 2.10 is met.

As a result, we can always specify (for all observations j) μ_j such that $\infty > \mu_j > 0$ to obtain consistency with Definition 2.1. We thus conclude that our specification of the personalized prices and quantities (i.e. the 'situation-dependent dictatorship' solution) for establishing the sufficiency result in Proposition 2.10 is consistent with the requirement that $\infty > \mu_j > 0$ in Definition 2.1.

Chapter 3

Modeling collective rationality: nonparametric tests on real-life data

Abstract

We nonparametrically test a general collective consumption model with public consumption and externalities inside the household. We further propose a novel approach to model special cases of the general collective model. These special cases include alternative restrictions on the 'sharing rule' that applies to each household, and that defines the distribution of the household budget over the household members. Our application uses data from the Russia Longitudinal Monitoring Survey (RLMS); the panel structure of this data set allows nonparametric testing of the behavioral models without relying on preference homogeneity assumptions across similar individuals. This application includes test results but also a power analysis for different specifications of the collective consumption model. Our main findings are that the most general collective model, together with a large class of special but still fairly general cases, cannot be rejected by the data, while other, restricted, versions of the general model, including the unitary alternative, are rejected. Since these tests are entirely nonparametric, this provides strong evidence in favor of models focusing on intrahousehold decision-making.[1]

[1] This chapter is adapted from Cherchye, De Rock and Vermeulen (2007c). We are grateful to André Decoster for useful comments and for giving an introduction to the RLMS. We also thank seminar participants in Leuven, London, Tilburg, Toulouse and at the 2006 European Meeting of the Econometric Society in Vienna for useful discussions.

3.1 Introduction

It is clear that a realistic modeling of the household consumption process should account for (partial) public consumption of certain commodities (e.g., rent and car use) and externalities (e.g., related to clothing) within the household. Using the tests presented in Chapter 2 we can nonparametrically test such a general consumption model without requiring any information on the intrahousehold allocation of the observed consumption bundle.

Nonparametric tests of collective models have been very scarce up to now. Snyder (2000) tests Chiappori's (1988) labor supply model with egoistic agents and observed labor supply. Using semi-algebraic theory for quantifier elimination, she develops a strict necessary and sufficient test for data consistency with collective rationality. Her tests however (only) applies for data sets with two observations. Cherchye and Vermeulen (2007) test unitary and collective models by directly starting from the respective $GARP$ conditions. Their procedure can be applied in settings with T observations but, again, the study focuses on the restrictive setting of labor supply behavior of egoistic individuals.

As for the practical implementation of the general collective rationality restrictions, it is to be recognized that the basic tests derived in Chapter 2 may be computationally burdensome if there are many observations. Still, as we will show in Section 3.2, some basic theoretical insights enable considerable efficiency gains in practical applications. The derivation and application of efficient testing algorithms constitutes a first objective of this chapter.

To the best of our knowledge, general collective models with public goods and/or externalities have not yet been tested nonparametrically on real-life data. In Section 3.3 we fill this gap. More specifically, we test the necessary condition in Proposition 2.7 on data that is drawn from the Russia Longitudinal Monitoring Survey (RLMS). The RLMS is one of the few surveys that enables constructing a very detailed panel of household consumption. This is interesting, since it permits nonparametric tests without having to assume that preferences are homogenous across similar individuals (or, in the unitary case, households). Moreover, although our sample covers a time series of only eight observations, there is enough relative price variation over time to test behavioral models in a meaningful way.

If the general collective model cannot be rejected, a further step may consist of testing more restrictive versions of the collective model. Evidently, such a more restrictive model implies a higher probability of rejection. Usually, restrictions to the general collective model are defined with respect to individual preferences or to the observability of certain intrahousehold allocations. An example is Chiappori's (1988, 1992) collective labor supply model with egoistic preferences and observed individual labor; see also our application in Chapter 4. In this chapter, we propose a novel approach to model restricted versions of the general collective model. Specifically, we consider the possibility of including alternative positions regarding the 'sharing rule' that applies to each household; this sharing rule defines the within-household distribution of the household budget, so reflecting the intrahousehold bargaining power of the different household members.[2] In Section 3.4 we derive the results that will enable us to nonparametrically test plausible but more restrictive alternatives of the general collective consumption model.

Finally, one potential drawback of a collective consumption model that takes into account externalities and public consumption inside the household, is that its generality makes it hardly rejectable. Although Proposition 2.8 shows that such a model can be rejected on the basis of couples' data when there are at least three commodities and three observations, the question remains how powerful the theoretical implications are in real-life empirical applications. Therefore, in addition to the nonparametric tests, we also include a power analysis of the various specifications of the collective consumption model. Such an analysis focuses on the probability of detecting an alternative hypothesis (e.g., based on Becker's (1962) notion of irrational behavior) to the detriment of the behavioral model under study. See also Bronars (1987), who introduced power assessment tools for the nonparametric unitary ($GARP$) condition; Andreoni and Harbaugh (2006) provide a survey of nonparametric power assessment tools that are currently available. Section 3.5 then contains the empirical results for alternative specifications of the collective consumption model, with a special focus on the power of the different specifications. Section 3.6 concludes.

[2] See Browning, Chiappori and Lewbel (2006) for a discussion of this sharing rule concept in a parametric treatment of the collective consumption model.

3.2 Algorithms for testing the general collective model

In this section we describe (basic) algorithms for testing the Propositions 2.7 and 2.10. As we will argue, these algorithms could be time consuming and therefore we also describe some efficiency enhancing mechanisms in subsection 3.2.2.[3]

3.2.1 Testing algorithms

We first present an algorithm for checking the *necessary condition for a collective rationalization* of the set of observations S in Proposition 2.7. This (necessity) test checks the condition in Proposition 2.7 for each possible configuration of the hypothetical relations. More formally, for any couple of observations (i, j) for which $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$ we must hypothesize $\mathbf{q}_i\ H_0^1\ \mathbf{q}_j$ or $\mathbf{q}_i\ H_0^2\ \mathbf{q}_j$; this implies 3 possible scenarios for each couple (i, j): $\mathbf{q}_i\ H_0^1\ \mathbf{q}_j$, $\mathbf{q}_i\ H_0^2\ \mathbf{q}_j$ and $(\mathbf{q}_i\ H_0^1\ \mathbf{q}_j \wedge \mathbf{q}_i\ H_0^2\ \mathbf{q}_j)$. Subsequently, for any combination of the respective scenarios, we should construct relations $H^1$and H^2 consistent with rules (ii)-(vi) of Proposition 2.7. Finally, the necessity condition stated in Proposition 2.7 should be satisfied for at least one such construction of hypothetical relations in order not to reject the general collective consumption model.

Before presenting the outline of the algorithm that we implemented, we introduce some additional notation. First, we define the set

$$D_j = \{(\mathbf{p}_i; \mathbf{q}_i) \mid \mathbf{q}_i\ R_0\ \mathbf{q}_j\}.$$

Next, we use that every specification of the hypothetical relations H_0^1 and H_0^2 (and the corresponding transitive closures H^1 and H^2) defines the sets $(m = 1, 2)$

$$D_j^m = \{(\mathbf{p}_i; \mathbf{q}_i) \mid \mathbf{q}_i\ H_0^m\ \mathbf{q}_j\} \text{ and } ID_j^m = \{(\mathbf{p}_i; \mathbf{q}_i) \mid \mathbf{q}_i\ H^m\ \mathbf{q}_j\}.$$

The following algorithm will be expressed in terms of the sets D_j^m and ID_j^m rather than the relations H_0^m and H^m. It goes as follows:

[3] In Cherchye, De Rock, Sabbe and Vermeulen (2007) we discuss more efficient algorithms, based on integer and linear programming techniques, which are capable to deal with larger data sets. Basically, these authors focus on an efficient (implicit) enumeration of all possible configurations of the hypothetical relations. In Section 3.3 we show that in our application the basic algorithm is sufficient.

Algorithm for necessity test

Step 1: For all $j \in \{1, \ldots, T\}$: construct the set D_j and set $C_j = \emptyset$. [Each set C_j captures all possible specifications of the sets D_j^1 and D_j^2 or, equivalently, the relations H_0^1 and H_0^2 that the algorithm considers in the successive iterations.]

Step 2: [See rule (i) in Proposition 2.7.] For all $j \in \{1, \ldots, T\}$: construct (D_j^1, D_j^2) such that: (a) $D_j^m \subseteq D_j$ $(m = 1, 2)$, (b) $D_j^1 \cup D_j^2 = D_j$ (c) $(D_j^1, D_j^2) \notin C_j$. If for any j such (D_j^1, D_j^2) does not exist, then STOP algorithm: *a collective rationalization of the set S is impossible.*

Step 3: [See rule (ii) in Proposition 2.7.] $\forall j \in \{1, \ldots, T\}$: construct (ID_j^1, ID_j^2) using Warshall's algorithm (Varian, 1982).

Step 4: For $j = 1, \ldots, T$ verify rule (iii) in Proposition 2.7: if OK, then go to $j+1$, unless $j = T$ then go to Step 5; else (a) $C_j = C_j \cup (D_j^1, D_j^2)$, (b) go to Step 2.

Step 5: For $j = 1, \ldots, T$ verify rule (iv) Proposition 2.7: if OK, then go to $j+1$, unless $j = T$ then go to Step 6; else (a) $C_j = C_j \cup (D_j^1, D_j^2)$, (b) go to Step 2.

Step 6: For $j = 1, \ldots, T$ verify rules (v) and (vi) in Proposition 2.7 for the constructed (ID_j^1, ID_j^2): if OK, then go to $j + 1$, unless $j = T$ then STOP algorithm: *the set S meets the necessary condition for a collective rationalization*; else (a) $C_j = C_j \cup$ (D_j^1, D_j^2), (b) go to Step 2.

This algorithm is finite in nature and is of order $3^{|D_1|+|D_2|+\cdots+|D_T|}$. Specifically, for any $(\mathbf{p}_i; \mathbf{q}_i) \in D_j$ we must (maximally) consider three possibilities: $(\mathbf{p}_i; \mathbf{q}_i) \in D_j^1$, $(\mathbf{p}_i; \mathbf{q}_i) \in D_j^2$ and $(\mathbf{p}_i; \mathbf{q}_i) \in D_j^1 \cap D_j^2$; for each $j \in \{1, \ldots, T\}$ this gives us $3^{|D_j|}$ possible specifications of the sets D_j^m. Since $3^{|D_1|+|D_2|+\cdots+|D_T|} \leq 3^{T^2}$, for T observations, we obtain a finite upper bound for the number of specifications to be checked. Note that this upper bound 3^{T^2} only applies if $D_j = S$ for all observations j, which is of course an extreme scenario.

We next consider the *sufficient condition for a collective rationalization* of the set of observations S in Proposition 2.10. This condition can be checked by means of the following algorithm:

Algorithm for sufficiency test

Step 1 For the given set S, define $S^* = \{(S^1, S^2)|\ S^1 \subseteq S \text{ and } S^2 = S \backslash S^1\}$. [The set S^* captures all possible specifications of S^1 and S^2.]

Step 2 For $(S^1, S^2) \in S^*$ verify $GARP$ for S^1 and S^2 (separately): if OK for some $(S^1, S^2) \in S^*$, then STOP algorithm: *a collective rationalization of the set S is possible*; if not OK for any $(S^1, S^2) \in S^*$, then STOP algorithm: *the set S does not meet the sufficient condition for a collective rationalization.*

Again, this algorithm is finite in nature: we maximally have to consider all possible subsets of S, which is exactly of magnitude 2^T for T observations.

3.2.2 Efficiency enhancing mechanisms

Given all this, the above algorithm implies checking the necessity condition (Proposition 2.7) for at most 3^{T^2} configurations of the hypothetical relations. Although this is an extreme scenario, also other more realistic scenarios may effectively entail a huge computational burden if there are many observations. This may be problematic even for present-day computers. We next present a procedure that may considerably enhance the efficiency of the necessity tests. Essentially, starting from the unitary $GARP$ test, this procedure constructs mutually independent subsets of observations for which the necessity condition can be tested separately.

As will be clear, this same procedure can also be applied to the algorithm for the sufficiency test (Proposition 2.10). Given that this sufficiency test is based on 'only' 2^T configurations, we focus below on the more cumbersome necessity condition.

Finally, besides applying the efficiency enhancing mechanisms, one can also successively apply the existing algorithms. For each subset of, say, k ($\leq T$) observations one can exploit that a collective rationalization is possible for the first l ($\leq k$) observations only if it is possible for the first $l-1$ observations. Hence, one may successively apply the testing algorithms to larger l (starting from $l = 3$), while each time respecting the feasibility restrictions associated with the (preceding) $l-1$ case (i.e. regarding possible specifications (D_j^1, D_j^2) for the necessity test and (S^1, S^2) for the sufficiency test). Given that in our applications

T was not too large (i.e. T=8, see Section 3.3.1), we did not use this insight to obtain the results in the next sections.

Unitary GARP testing

As a preliminary step, we recall that the standard (unitary) *GARP* test (see Definition 1.3) starts from the R_0 relations, which subsequently form the basis for constructing (unitary) revealed preference relations (via Warshall's algorithm; see Varian, 1982). The *GARP* condition then states that each observation j should be cost minimizing over its revealed preferred bundles.

Our efficiency enhancing procedure concentrates on the *GARP* violating condition for a couple of observations $(i,\, j)$, *i.e.*

$$\mathbf{q}_i \; R \; \mathbf{q}_j \text{ and } \mathbf{p}_j'\mathbf{q}_j > \mathbf{p}_j'\mathbf{q}_i. \tag{3.1}$$

If (3.1) is not met, the couple (i, j) cannot be involved (at the level of the individual household members) in a rejection of the necessity condition for a *collective rationalization* of the data.[4] Specifically, in such a case each constellation of the member-specific hypothetical relations H^1 and H^2 consistent with the rules (i)-(iv) in Proposition 2.7 will never imply a violation of the closing conditions (v) and (vi) of Proposition 2.7 that involves i and j. This is obtained by noting that $\mathbf{q}_i \; H^m \; \mathbf{q}_j$ ($m = 1$ or 2) only if $\mathbf{q}_i \; R \; \mathbf{q}_j$, which in turn entails $\mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'\mathbf{q}_i$ given that (3.1) is not met.

Filtering

The first step of the procedure 'filters out' from the original data set the observations that are, as we will explain, 'irrelevant' for the necessity test. Specifically, it uses the above insight to concentrate exclusively on couples of observations (i, j) that satisfy (3.1). Of course, given the construction of the member-specific hypothetical relations H^1 and H^2, we should also include the sequence(s) of observations k that lie between i and j (i.e. $\mathbf{q}_i \; R \; \mathbf{q}_k$ and $\mathbf{q}_k \; R \; \mathbf{q}_j$). More generally, for each couple (i, j) we define the set

$$Seq(i, j) = \{k \mid \mathbf{q}_i \; R \; \mathbf{q}_k \; and \; \mathbf{q}_k \; R \; \mathbf{q}_j\} \text{ if } (3.1),$$
$$Seq(i, j) = \emptyset \text{ if not } (3.1).$$

[4] We note that the order of the *GARP* violating couple (i, j) is relevant. Specifically, the empirical content of the condition $(\mathbf{q}_i \; R \; \mathbf{q}_j \wedge \mathbf{p}_j'\mathbf{q}_j > \mathbf{p}_j'\mathbf{q}_i)$ is clearly different from that of $(\mathbf{q}_j \; R \; \mathbf{q}_i \wedge \mathbf{p}_i'\mathbf{q}_i > \mathbf{p}_i'\mathbf{q}_j)$.

It follows from our above argument that we may concentrate on the union of the sets $Seq(i,j)$; we further refer to that union as *Useq*. Intuitively, this means focusing on the couples of observations i and j (and the associated 'in between' observations k) that cannot be rationalized by the unitary model.

The observations that do not belong to some $Seq(i,j)$ as defined above become 'irrelevant' for the necessity test. Given the exponential increase of the number of computations needed to test collective rationality for larger data sets, excluding these observations may considerably shorten the time needed to reach a verdict. In fact, $GARP$ consistency of a particular sample means that all observations are 'filtered out' by construction. In that case, all observations become irrelevant for the collective necessity test, meaning that the test itself becomes redundant.

Subset testing

The second step, which we refer to as 'subset testing', partitions the (filtered) data set *Useq* into subsets that are mutually independent when testing the necessity condition. In this context, 'mutual independence' indicates that for any two subsets, say $Useq^1$ and $Useq^2$ (with $\bigcup_{l=1,2} Useq^l \subseteq Useq$ and $\bigcap_{l=1,2} Useq^l = \emptyset$), we have that $Useq^1$ does not include observations that are implicated in some $GARP$ violation contained in the subset $Useq^2$, and *vice versa*. Formally, this means that for each combination of couples $(i_1, j_1) \in Useq^1 \times\ Useq^1$ and $(i_2, j_2) \in Useq^2 \times\ Useq^2$, we have $i_l, j_l \notin Seq(i_m, j_m)$ $(l, m \in \{1, 2\},\ l \neq m)$.

Indeed, a similar argument as before implies that we may restrict to testing the necessity condition for a *collective rationalization* of the data at the level of the separate subsets rather than the (unpartitioned) union *Useq*. Again, this subset testing may considerably reduce the computational burden of the necessity test, especially when the number of mutually independent subsets gets large.

To conclude, we remark that the partitioning of *Useq* can proceed efficiently by starting from the sets $Seq(i,j)$. Specifically, using the earlier definitions, it can be imposed that for any $(i_1, j_1) \in Useq^1 \times\ Useq^1$ and $(i_2, j_2) \in Useq^2 \times\ Useq^2 : Seq(i_1, j_1) \cap Seq(i_2, j_2) = \emptyset$. Evidently, one should then only focus on the $GARP$ violating couples of observations (which satisfy condition (3.1)). As a result, a simple enumeration algorithm, which consecutively considers the different violations of the unitary *GARP* test, can identify the maximum number of independent subsets of *Useq*.

3.3 Empirical application of the necessity tests for collective rationality

This section presents the empirical results for our application of the necessity tests to data taken from the Russia Longitudinal Monitoring Survey (RLMS). Before presenting these results, we discuss some particularities of the RLMS data set.

3.3.1 Data

The data are drawn from the RLMS. More specifically, they come from Phase II of the RLMS, which covers the time period between 1994 and 2003 (Rounds V-XII). The RLMS contains a lot of socioeconomic information like detailed expenditures, incomes, assets and health from a nationally representative sample of Russian households. It was designed to measure the impact of Russian reforms on the economic well-being of households and individuals. Although the RLMS survey design focuses on a longitudinal study of populations of dwelling units, it allows a panel analysis of those households remaining in the original dwelling unit over time.

The sample selection for the present study is for households consisting of a couple only. We select households where both members are employed to mitigate the issue of the non-separability between consumption and leisure (see Browning and Meghir, 1991). Finally, we only consider households that were observed in all the available rounds of Phase II of the RLMS. This results in a sample of 148 couples that are observed eight times.

Using the same criteria we also selected a sample of 108 single households, consisting of 9 males and 99 females. Given that economic theory predicts consistency with *GARP* for this sample, it will be used to perform a validity check of our *GARP* test.

In our empirical exercises, we focus on a fairly detailed commodity bundle that consists of 21 nondurable goods: (1) bread, (2) potatoes, (3) vegetables, (4) fruit, (5) meat, (6) dairy products, (7) fat, (8) sugar, (9) eggs, (10) fish, (11) other food items, (12) alcohol, (13) tobacco, (14) food outside the home, (15) clothing, (16) car fuel, (17) wood fuel,

(18) gas fuel, (19) luxury goods, (20) services and (21) rent. Although the disaggregation of food items may appear far too detailed, it should be noted that the average budget share of food equals 40% for the selected sample (see also the Appendix). Prices are obtained by averaging recorded prices across the households in a given census region. Some of the commodities that we use are aggregate commodities. The price index for a composite commodity is the weighted geometric mean of the prices of the different items in the aggregate good, with weights equal to the average budget shares in a given census region (the Stone price index). Some summary statistics for our sample are given in the Appendix.

Anticipating the empirical results, it should be stressed that we apply the nonparametric collective rationality test to each separate household, which implies that each household's quantity and price observations form a separate set S. As shown in Chapter 2, falsification of the general collective model requires (at least) three commodities and three observations. Hence, given that each household is observed eight times and the commodity bundle consists of more than three goods, the general collective model is potentially rejectable. Another advantage of testing at the household level is that we do not need to rely on possibly controversial preference homogeneity assumptions across individuals in different households. Of course, given that we focus on a static model, the results are still vulnerable for heterogeneity through time such as, e.g., habits formation (see Crawford, 2007, for nonparametric results for the unitary model).

3.3.2 Empirical results

As a preliminary step, we verified if the $GARP$ test is a good starting point for our analysis of rationality. Assuming individual rationality for single households, the economic theory predicts that these households satisfy $GARP$. From Table 3.1 we conclude that this indeed holds for the single households in the RLMS.

Table 3.1. *GARP* test results for single households

	Frequency	*Percentage*
GARP rejected	0	0.00
GARP not rejected	108	100.00

Table 3.2 summarizes the empirical results for the necessity test of the general collective consumption model. It is clear from the upper panel of the table that all couples in our sample pass the necessity test. From the middle panel, it can be deduced that the consumption behavior of 117 couples (79% out of the 148 couples) can be described by means of a unitary model. In other words, their set of observed quantity-price bundles satisfies the *GARP* condition. Following our filtering procedure, all eight observations of these households are irrelevant for the necessity test in the sense described above. Note that the 21% rejections of the unitary model show that there is enough price variation to test behavioral models.

Next, all households that do not pass the *GARP* test have at least three irrelevant observations. In fact, most of these households have five or six irrelevant observations, which considerably favors the efficiency of the necessity test algorithm. This indicates that the suggested filtering procedure may considerably enhance the efficiency of the testing algorithm in practical applications.

The results of the subset testing procedure are given in the bottom panel of Table 3.2. For most households that do not satisfy the *GARP*, only a single subset can be created from the original data sets. In such cases, all 'relevant' observations are linked to each other via revealed preference relationships, which makes a separate analysis of subsets impossible. For one household, we can distinguish two subsets for which the necessity conditions can be tested separately. It has five relevant observations, which are allocated to subsets with, respectively, two and three observations. More generally, one may expect this subset procedure to be particularly useful for larger data sets.

What can we infer from these results? A first conclusion is that the general collective consumption model seems to provide an adequate description of the observed consumption behavior of the couples in our sample, at least if the evaluation criterion is non-rejection of its theoretical implications when tested on real data. The unitary model's *GARP* condition, on the other hand, is rejected for 21% of the households in our sample. In view of the fact that we only have eight observations per household, this gives fairly strong evidence in favor of the collective approach. Moreover, we performed the nonparametric tests at the individual household level, which excludes the interpretation of the *GARP* violations as revealing cross-sectional unobserved heterogeneity. Finally, we also recall that *GARP* was not rejected for households with only one member.

According to these unitary results, we have to note that we implemented a 'basic' *GARP* test, meaning that we did not control for measurement error. That is, we assumed that the observed prices and quantity are exactly the ones that the individuals face; as such we ignored for instance misreporting of the price-quantity data.[5] Of course, one could argue that rejections of *GARP* can be explained by these measurement errors and that therefore the number of rejections of the unitary model is actually lower. Indeed, it could be that in reality a given household satisfies *GARP*, but that because of measurement error we reject it. See, e.g., Varian (1985) for a more thorough discussion of integrating measurement error in nonparametric tests.
Clearly, integrating measurement error would enrich our results and for the unitary model there are several methods to do this; see, e.g., Blundell, Browning and Crawford (2003, 2007) and Crawford (2007) for recent applications of the idea of Varian (1985). However, to the best of our knowledge, there are currently no similar results for the collective model. Therefore in order to compare the results for our unitary and collective tests, we opted in this chapter to ignore measurement error in both settings. Tackling this problem, is work for the near future. In this respect, Cherchye, De Rock, Sabbe and Vermeulen (2007) discuss the possibility to use their IP formulation for dealing with measurement errors in practical applications; i.e. they show that the IP formulation can be adapted to include the idea of Varian (1985) for dealing with such errors.

Another conclusion may be that the theoretical implications of the general collective consumption model are simply too 'generous' to obtain violations from realistic data. This alternative interpretation motivates our next section, which discusses how far we can go in restricting the general model. The empirical assessment in Section 3.5 also includes a power analysis of the restricted collective consumption model, which

[5] As is standard in the literature, measurement error only refers to wrongly observed prices and/or quantities

should give a deeper insight into the effective 'generosity' of the alternative model specifications.

Finally, and perhaps most importantly, the following empirical analysis focuses on sufficiency conditions for collective rationality, which naturally complements this first-step assessment of the necessity conditions. In particular, while the results in Table 3.2 imply that we cannot exclude a collective rationalization of the data, these further sufficiency results will reveal whether or not it is *certainly* feasible to define (restricted) collective consumption models that rationalize the observed couples' behavior.

Table 3.2. Necessity test results for couples

	Frequency	*Percentage*
Necessity test		
CR rejected	0	0.00
CR not rejected	148	100.00
Number of irrelevant observations		
0	0	0.00
1	0	0.00
2	0	0.00
3	1	0.68
4	1	0.68
5	8	5.41
6	21	14.19
7	0	0.00
8	117	79.05
Number of subsets (of relevant observations)		
0	117	79.05
1	30	20.27
2	1	0.68

3.4 Restricting the general collective model: a new approach

Given that in the previous section we were not able to reject the general collective model, one may investigate the extent to which more restrictive models are equally plausible. This may be an interesting research question from an empirical point of view. For example, it may examine the validity of (frequently employed) restrictions with respect to individual preferences or to the observability of certain intrahousehold allocations.

This section proposes a novel way to define restrictions on the general collective model. Specifically, the restrictions directly constrain the *sharing rule* that applies within each household. After introducing some general concepts, we present operational sufficient conditions that enable testing data consistency with collective rationality under alternative specifications of the sharing rule restrictions. As we will indicate, these sufficiency tests actually boil down to verifying the unitary *GARP* condition on a transformed data set. This suggests the consumption models that underlie the sufficiency tests as direct collective extensions of the unitary model.

3.4.1 Collective rationality under sharing rule restrictions

We suggest an approach that is as general as possible with respect to the structure of individual preferences and the non-observability of intrahousehold allocations. The approach is directly based on the decentralization result discussed in Chapter 2, which essentially implies that observed household consumption results from a two-step allocation procedure. In the first step, individuals divide the household's total expenditures among each other. In the second step, each individual allocates her or his expenditure share to the household's decomposed (private and public) consumption bundles. An important difference with the more usual decentralization result (that applies under egoistic preferences), is that this (individual) consumption may encompass not only one's own private consumption, but also the partner's private consumption and public consumption. Given the assumption of Pareto-efficiency, this implies that the intrahousehold allocation process involves personalized (or Lindahl) prices, which add up to observed prices.

Suppose that for each member we observe the *true* personalized quantities $\mathbf{q}_j^1$, $\mathbf{q}_j^2$ and $\mathbf{q}_j^h$ and the *true* personalized prices $\mathbf{p}_j^1$, $\mathbf{p}_j^2$ and $\mathbf{p}_j^h$. Formally, using (2.1) and (2.2), this means that we can specify $\widehat{\mathbf{q}}_j = (\mathbf{q}_j^1, \mathbf{q}_j^2, \mathbf{q}_j^h)$, $\widehat{\mathbf{p}}_j^1 = (\mathbf{p}_j^1, \mathbf{p}_j^2, \mathbf{p}_j^h)$ and $\widehat{\mathbf{p}}_j^2 = (\mathbf{p}_j - \mathbf{p}_j^1, \mathbf{p}_j - \mathbf{p}_j^2, \mathbf{p}_j - \mathbf{p}_j^h)$. Then by the decentralization result, we can define for each household member the respective individual budget shares.

Definition 3.1. *Let S be a set of observations and $\mathbf{q}_j^1, \mathbf{q}_j^2, \mathbf{q}_j^h$ and $\mathbf{p}_j^1, \mathbf{p}_j^2, \mathbf{p}_j^h$ be the true observed personalized quantities and prices. Then for each observation j: $\eta_j = \frac{(\widehat{\mathbf{p}}_j^1)'\widehat{\mathbf{q}}_j}{\mathbf{p}_j'\mathbf{q}_j}$ equals the share of individual 1 and $\frac{(\widehat{\mathbf{p}}_j^2)'\widehat{\mathbf{q}}_j}{\mathbf{p}_j'\mathbf{q}_j} = 1 - \eta_j$ equals the share of individual 2, with $\widehat{\mathbf{q}}_j = (\mathbf{q}_j^1, \mathbf{q}_j^2, \mathbf{q}_j^h)$, $\widehat{\mathbf{p}}_j^1 = (\mathbf{p}_j^1, \mathbf{p}_j^2, \mathbf{p}_j^h)$ and $\widehat{\mathbf{p}}_j^2 = (\mathbf{p}_j - \mathbf{p}_j^1, \mathbf{p}_j - \mathbf{p}_j^2, \mathbf{p}_j - \mathbf{p}_j^h)$.*

Individual m's share thus equals the ratio of that individuals' expenditures on the true personalized consumption bundle, valued at true personalized prices, to the household's total expenditures. In general, however, we do not observe the true personalized quantities or personalized prices. Therefore, we replace them by the feasible personalized quantities (respectively prices) introduced in (2.1) (respectively (2.2)), which obtains 'feasible shares'.

The rationality condition for the restricted collective consumption model essentially includes restrictions for the so-called sharing rule, which specifies the individuals' shares in Definition 3.1. (This sharing rule may be interpreted as reflecting the bargaining power of the different household members in the household allocation process; see in particular the duality result in Proposition 3 of Browning, Chiappori and Lewbel, 2006.) More specifically, a broad class of special cases of the general collective model can be defined through alternative sharing rule restrictions of the form of $\alpha(\mathbf{p}_j'\mathbf{q}_j) \leq (\widehat{\mathbf{p}}_j^m)'\widehat{\mathbf{q}}_j \leq (1-\alpha)(\mathbf{p}_j'\mathbf{q}_j)$. This effectively imposes that each individual receives a budget share of at least $\alpha \in [0, 0.5]$. For example, α can then be interpreted as a minimum requirement for both individuals to prevent the dissolution of the couple.

To avoid any possible confusion, we stress that restrictions on sharing rules do not imply any specific assumption regarding the true (unobserved) values of the personalized quantities or prices, but only regarding their product. More formally, it is easy to verify that, for any given share η and any given personalized quantities $\widehat{\mathbf{q}}$ (or, conversely, $\widehat{\mathbf{p}}^1$and $\widehat{\mathbf{p}}^2$) one can always find personalized prices $\widehat{\mathbf{p}}^1$and $\widehat{\mathbf{p}}^2$ (or $\widehat{\mathbf{q}}$) such that $\eta = \frac{(\widehat{\mathbf{p}}^1)'\widehat{\mathbf{q}}}{\mathbf{p}'\mathbf{q}}$ and 1− $\eta = \frac{(\widehat{\mathbf{p}}^2)'\widehat{\mathbf{q}}}{\mathbf{p}'\mathbf{q}}$.[6]

[6] Note that this does not exclude that these extra restrictions on the sharing rule imply extra restrictions on the primitives of our model (i.e. on the individual preferences and the bargaining weights). As such, our results hereafter, also implicitly test these extra assumptions.

Based on Definition 2.1, the condition for a collective rationalization of a set of observations S under the additional sharing rule restrictions $\alpha(\mathbf{p}_j'\mathbf{q}_j) \leq (\widehat{\mathbf{p}}_j^m)'\widehat{\mathbf{q}}_j \leq (1-\alpha)(\mathbf{p}_j'\mathbf{q}_j)$ is defined as follows.

Definition 3.2. *Let* $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \ldots, T\}$ *be a set of observations and* $\alpha \in [0, 0.5]$. *A pair of utility functions* U^1 *and* U^2 *provides an* α*-restricted collective rationalization* (α-CR) *of the observed set* S *if for each observation* j *there exist feasible personalized quantities* $\widehat{\mathbf{q}}_j = (\mathfrak{q}_j^1, \mathfrak{q}_j^2, \mathfrak{q}_j^h)$ *and* $\mu_j \in \mathbb{R}^n_{++}$ *such that*

$$
\begin{aligned}
&(i) \;\; \alpha(\mathbf{p}_j'\mathbf{q}_j) \leq (\widehat{\mathbf{p}}_j^1)'\widehat{\mathbf{q}}_j \leq (1-\alpha)(\mathbf{p}_j'\mathbf{q}_j);\\
&(ii) \; U^1(\widehat{\mathbf{q}}_j) + \mu_j U^2(\widehat{\mathbf{q}}_j) \geq U^1(\widehat{\mathbf{z}}) + \mu_j U^2(\widehat{\mathbf{z}})
\end{aligned}
$$

for all $\widehat{\mathbf{z}} = (\mathfrak{z}^1, \mathfrak{z}^2, \mathfrak{z}^h)$ *with* $\mathfrak{z}^1, \mathfrak{z}^2, \mathfrak{z}^h \in \mathbb{R}^n_+$ *and* $\mathbf{p}_j{}'(\mathfrak{z}^1 + \mathfrak{z}^2 + \mathfrak{z}^h) \leq \mathbf{p}_j'\mathbf{q}_j$.

The interpretation is directly analogous to that of Definition 2.1.[7] The mere difference is the sharing rule restriction that is included in *(i)*. As indicated above, such a restriction may be motivated by dissolution-preventing arguments in practical applications. Still, it is worth stressing that it also encompasses a multitude of other special cases, including the more standard restrictions that are defined with respect to (possibly egoistic) individual preferences or the observability of certain intrahousehold allocations.

More generally, one may use alternative sharing rule restrictions for different household observations, where these restrictions may vary depending on the household and the specific situation under consideration. We will not explicitly consider such variants in this study, but our following discussion is easily extended to include such cases. Such extensions may for instance be worthwhile to consider in applications where additional prior information regarding the intrahousehold process is available, or when sharing rule recovery forms a main purpose of the testing exercise.

3.4.2 Sufficiency conditions for collective rationality

Contrary to Section 3.2, we exclusively focus on sufficient collective rationality conditions in the following. Consistency with the sufficiency

[7] Because of the decentralization result discussed before rule (ii) is equivalent with the existence of personalized prices $\widehat{\mathbf{p}}_j^m = (\mathfrak{p}_j^1, \mathfrak{p}_j^2, \mathfrak{p}_j^h)$ such that (a) $U^1(\widehat{\mathbf{q}}_j) \geq U^1(\widehat{\mathbf{z}})$ for all $\widehat{\mathbf{z}} = (z^1, z^2, z^h)$ with $z^1, z^2, z^h \in \mathbb{R}^n_+$ and $(\widehat{\mathbf{p}}_j^1)'(\mathfrak{z}^1 + \mathfrak{z}^2 + \mathfrak{z}^h) \leq (\widehat{\mathbf{p}}_j^1)'\widehat{\mathbf{q}}_j$ and (b) $U^2(\widehat{\mathbf{q}}_j) \geq U^2(\widehat{\mathbf{z}})$ for all $\widehat{\mathbf{z}} = (z^1, z^2, z^h)$ with $z^1, z^2, z^h \in \mathbb{R}^n_+$ and $(\widehat{\mathbf{p}}_j^2)'(\mathfrak{z}^1 + \mathfrak{z}^2 + \mathfrak{z}^h) \leq (\widehat{\mathbf{p}}_j^2)'\widehat{\mathbf{q}}_j$.

condition (for particular α) means that there *certainly* exists at least one specification of the intrahousehold allocation that guarantees consistency of observed behavior with collective rationality as defined in Definition 3.2.[8] Building further on Proposition 2.10, we get the following sufficiency condition for an α-*CR* of the data.[9]

Proposition 3.3. *A* sufficient condition *for the existence of utility functions* U^1 *and* U^2 *that provide an* α-CR *of the observed set* S *is that there exists a partitioning* N_1, N_2 $(N_1 \cup N_2 = \{1, \ldots, T\}; N_1 \cap N_2 = \emptyset)$ *such that*
(i) $\forall j \in N_1 : \mathfrak{q}_j^1 = \alpha \mathbf{q}_j,\ \mathfrak{q}_j^2 = (1-\alpha)\mathbf{q}_j$ *and* $\mathfrak{q}_j^h = \mathbf{0}$;
(ii) $\forall j \in N_2 : \mathfrak{q}_j^1 = (1-\alpha)\mathbf{q}_j,\ \mathfrak{q}_j^2 = \alpha \mathbf{q}_j$ *and* $\mathfrak{q}_j^h = \mathbf{0}$*;*
(iii) $\forall j \in S : \mathfrak{p}_j^1 = \mathbf{p}_j,\ \mathfrak{p}_j^2 = \mathfrak{p}_j^h = \mathbf{0}$;
(iv) $\{(\widehat{\mathbf{p}}_j^1, \widehat{\mathbf{q}}_j); j = 1, \ldots, T\}$ *and* $\{(\widehat{\mathbf{p}}_j^2, \widehat{\mathbf{q}}_j); j = 1, \ldots, T\}$ *both satisfy the* GARP.

Clearly the solution in this proposition is a technical one. Therefore we stress once more that if the data satisfy Proposition 3.3, then this does not imply that other (more realistic) solutions are excluded. It only states that there is certainly one solution that satisfies the conditions of Definition 3.2. For example, it could be that the preferences are not egoistic, but if the data satisfies Proposition 3.3 then we cannot exclude egoism. Indeed the proposed solution of personalized prices in Proposition 3.3, shows that each individual pays for his own consumption (recall that, e.g., externalities of member 1 are captured by $\mathfrak{p}_j^2 \neq \mathbf{0}$). We refer to Chapter 4 for a more thorough discussion of the ‘egoistic model’.

One interpretation of this sufficiency condition is that it requires that the set S can be transformed into two sets S_1 and S_2 that both satisfy the *GARP*. More specifically, $S_1 = \{(\mathbf{p}_j; \alpha_j \mathbf{q}_j),\ j = 1, \ldots, T\}$ and $S_2 = \{(\mathbf{p}_j; (1-\alpha_j)\mathbf{q}_j),\ j = 1, \ldots, T\}$ where $\alpha_j = \alpha$ if $j \in N_1$ (*i.e.*, individual 1 receives the share α) and $\alpha_j = (1-\alpha)$ if $j \in N_2$ (i.e., individual 1 receives the share $(1-\alpha)$). To give an example, assume that α is equal to 0.3. In terms of Definition 3.2 , this means that each individual member gets at least 30 percent of the total household means. A

[8] One could equally derive a necessity condition for data consistency with the collective model for particular α. The derivation is easily analogous to that of Proposition 2.7. Still, given our main focus on sufficiency conditions in the following, we refrain from explicitly including this result here.

[9] Again this sufficient condition could be adapted to groups of commodities instead of whole bundles. See also footnote 14 in Chapter 2.

sufficient condition for such a collective rationalization to be possible is data consistency with the member-specific *GARP* conditions when the two household members receive 30 or 70 percent of the total household means. However, the specific value may vary depending on the specific observation. Consequently, for some observations an individual may receive a share of 70 percent, whereas it may amount to only 30 percent in other situations.

The nonparametric test for an α-*CR* first transforms the observed consumption bundles $\mathbf{q}_j$ $(j = 1, \ldots, T)$ to $\alpha_j \mathbf{q}_j$ and subsequently tests the standard *GARP* condition on the resulting sets S_1 and S_2. The intuition behind the result is that both individuals should maximize their utility subject to the shares that are allocated to them, and that their choices should be consistent across the observations, independently of the fact whether they received the share α or $(1 - \alpha)$. Of course, since intrahousehold allocations are assumed Pareto-efficient, the above requirements should be simultaneously satisfied for both individuals.

A few observations are in order with respect to the α-*CR* restrictions. First, if α equals 0.5, then the implications of the above restricted collective model reduce to the standard *unitary model*. Indeed, if all consumption bundles are multiplied by 0.5, then it is easily verified that the corresponding *GARP* tests for the individual members is formally equivalent to the unitary *GARP* test for the household.[10] As such, we cannot distinguish the 0.5-*CR* model from the unitary model. More generally, the empirical implementations of the unitary model coincide with those of the α-*CR* model if α is constant over all observations.

A second limiting case is the so-called *situation-dependent dictatorship* situation, which is described in Proposition 2.10. This model can be rationalized by setting α equal to zero, implying that an individual either has control of expenditures equal to the household's total resources, or controls no expenditures at all.

A final observation concerns the fact that the sufficiency collective rationality conditions (such as the ones in Proposition 2.10 and 3.3) are generally much easier to test than the necessity conditions (in Proposition 2.7). Specifically, independent of the chosen α, they require checking at most 2^T alternative specifications of the sets N_1 and N_2, which

[10] More precisely, given the specification of the personalized prices and quantities, $\widehat{\mathbf{p}}_j^m \widehat{\mathbf{q}}_j \geq \widehat{\mathbf{p}}_j^m \widehat{\mathbf{q}}_i$ if and only if $\mathbf{p}_i(0.5\mathbf{q}_i) \geq \mathbf{p}_i(0.5\mathbf{q}_j)$. Clearly this is also equivalent with $\mathbf{p}_i\mathbf{q}_i \geq \mathbf{p}_i\mathbf{q}_j$). So member m satisfies *GARP* if and only if the household as a whole satisfies *GARP*.

is much below the maximum number of 3^{T^2} configurations in the necessity tests. Again, further efficiency gains may be realized by various refinements of the testing algorithm (including filtering and subset testing). For the sake of compactness, we refrain from a detailed discussion here, but the treatment is analogous to that in Section 3.2. Also, our own application, including the computation of the power measures (which imply 1000 iterations for each household and for the different α-specifications under consideration), does not utilize such efficiency-enhancing strategies. Nevertheless, our different exercises required little computation time (e.g., for a given α the power assessment for the whole sample of all households only took a couple of minutes for a standard PC configuration).

3.5 Empirical application of the α-restricted tests for collective rationality

This section presents the results for α-restricted collective rationality tests when applied to our RLMS data set. As a main focus will be on the power of the alternative collective rationality models, we first outline our procedure for the power assessment.[11]

3.5.1 Power assessment method

Generally, a power analysis evaluates the probability of detecting an alternative hypothesis to the model under study. Bronars (1987) first defined power measures for the unitary model. His alternative hypothesis was based on Becker's (1962) notion of irrational behavior, which states that households randomly choose consumption bundles that exhaust the available budget. Bronars' power measures then capture the probability of rejecting the $GARP$ condition for such randomly drawn consumption bundles from the observed budget hyperplanes. In this chapter, our power assessment basically extends Bronars' (unitary)

[11] As we concentrate on sufficient conditions for α-restricted collective rationality, our power estimates may also be interpreted as 'upper bounds' for the power of necessary and sufficient (α-restricted) collective rationality conditions. One could similarly conceive 'lower bounds' for the power measures starting from operational necessity conditions for collective rationality. Like before, these lower bounds will lie closer to the upper bounds for α closer to their maximum value of 0.5.

procedure for the collective rationality tests, except from some modifications that specifically relate to the nature of our RLMS data. In Chapter 4, we introduce an alternative power measure.

At least two data features impact on the power assessment. First, as Bronars has illustrated, power measures crucially depend on the degree of relative price variation in the data. For example, if budget hyperplanes do not intersect for a particular data set, then the unitary model can never be rejected for this data. The results in Section 3.3 show that there is enough price variation in our sample for such rejection. Second, and more specific to our application, the power assessments should account for the presence of zero expenditures in the data. Generally, this is an important feature of microdata on detailed consumption, which is a particularly relevant consideration for the RLMS (where the data for each survey round refer to the consumption in a single week).

It should be noted that our focus on nondurables mitigates the zero expenditure problem to some extent. In addition, given the relative importance of food in the Russian consumption, the issue of zero expenditures on detailed food items due to infrequency of purchase is probably less important than in OECD countries.[12] Still, we do believe it is important to explicitly take up the presence of zero expenditures in our power assessment. In fact, without explicit correction, randomly drawing commodity bundles from a household's budget constraint obtains a zero probability of simulating zero consumption of a certain item. Clearly, such a simulation does not match reality if zero expenditures are effectively observed.

Given all this, we use a power assessment procedure that starts from Becker's (1962) irrational behavior, but takes into account the observed zero expenditures. More specifically, we first calculate per household h and per commodity i the proportion of strictly positive expenditures in the eight household observations. Let us denote this proportion by z_{hi}. The drawing of household-specific irrational commodity bundles then proceeds as follows. First, per commodity i and per time period t we draw a random number from the uniform distribution between 0 and 1. If this commodity- and time-specific number is greater than z_{hi}, then

[12] Also, our evaluation of the collective rationality conditions at the individual household level alleviates the potential problem of zero expenditures: if there are no expenditures on a given commodity in all eight rounds, then this household simply has a smaller consumption set than a household that has expenditures on all the commodities.

the number v_{hit} is set equal to zero. In the opposite case, the number v_{hit} is the result of a new drawing from the uniform distribution (between 0 and 1). Subsequently, the budget share w_{hit} for household h of commodity i at time t is defined as $(v_{hit}/\sum_i v_{hit})$. Finally, the random/irrational quantity bundle for household h at time t is obtained by multiplying the thus obtained vector of budget shares by the observed expenditure level (of household h at time t), and dividing the different components of the resulting vector by the corresponding components of the observed commodity price vector (for household h at time t).[13]

For each household and per RLMS-round, 1000 random consumption bundles are constructed in the way just described. The advantage of the procedure is that it results in an expected proportion of zero expenditures that complies with the observed proportion. Moreover, if a household does not have any expenditures on a particular commodity in all eight rounds of the RLMS, then it will never be randomly allocated a consumption bundle with strictly positive expenditures on that commodity.

The randomly constructed consumption bundles can now be used to estimate the power of the rationality tests associated with different collective consumption models. A power measure gives the probability that a particular collective rationality test detects such irrational (budget-exhausting) behavior.[14] Our empirical exercise specifically considers two power measures, which exploit the panel structure of our data set and provide useful complementary information. The first measure (labeled *Power 1*) captures the proportion of the 1000 random cases where Becker's irrational behavior is detected for at least one household in the sample. The underlying idea is that a behavioral model is rejected if not all households can be fit in its theoretical implications. However, it is well possible that an outlier-household completely determines this first power measure. Therefore, our second power measure (labeled *Power 2*) gives the average proportion of households where Becker's irrational

[13] This modeling of irrational behavior actually complies with Bronars' (1987) Method 2. Compared to his first method, where he uses budget shares that are uniformly distributed over the budget set, this method excludes *extreme* irrational behavior (and, hence, by construction implies lower power estimates).

[14] Remark that there may be some confusion of tongues when using the notion of *irrational behavior*. In our study, we use the term to refer to randomly drawn commodity bundles, and *not* to household behavior that cannot be fit in the unitary model, which may actually be consistent with a more general collective specification.

behavior is detected across all (1000) randomly drawn scenarios. Summarizing, the *Power 1* measure captures the power of the model at the level of the sample as a whole, while the *Power 2* measure provides complementary information regarding the power of the model at the level of the individual households.

3.5.2 Empirical results

Table 3.3 summarizes the test results associated with the α-restricted collective consumption models. Before discussing these results in greater detail, recall that our analysis focuses on sufficiency tests for collective rationality. As mentioned before, consistency with these sufficiency conditions for particular α means that there exists at least one definition of the collective consumption model (corresponding to specific sharing rule restrictions) that rationalizes the observed behavior.

A first observation then pertains to the case where α equals 0.50, which states that the two members divide the resources equally under all circumstances. As discussed before, the empirical implications of this collective model are indistinguishable from those of the unitary model. Given this, the 31 households that did not pass the *GARP* test (see our discussion of the necessity test results) can never meet the empirical conditions corresponding to this limiting case of the collective consumption model. This also appears in Table 3.3.

Next, we find in the table that all couples meet the other ('extreme') situation-dependent dictatorship condition (for $\alpha = 0$). This implies that there certainly exists a collective rationalization of the data for the general collective consumption model. To recall, in Section 3.3. we obtained that the necessary conditions for collective rationality are satisfied and here we construct a possible solution that satisfies the restrictions of the general collective model. Given this, one can then investigate which extra restrictions can be added to this general model. More precisely, here we regard to what extent the above findings change for alternative sharing rule constraints. Table 3.3 makes clear that lower α values result in less households not passing the associated rationality tests. For example, 19 couples do not satisfy the α-CR restrictions under $\alpha = 0.495$ (i.e., the couples' members receive either 49.5% or 50.5% of the total expenditures). This number steadily decreases towards zero for lower α: only a single couple violates the α-CR restrictions for $\alpha = 0.45$; and all households meet the sufficiency restrictions when α is not above 0.40.

These findings suggest that, even though the definition of the collective consumption models underlying the respective sufficient rationality conditions may seem restrictive to some, a wide range of such models is effectively able to describe the observed couples' consumption behavior. Interestingly, these favorable test results should not necessarily be attributed to a low power of the different α-*CR* models: the *Power 1* values above 90% for all the models where α is at least equal to 0.45; and it equals no less than 67% for the model that uses α equal to 0.40, which -to recall- cannot be rejected for any couple in our sample.

As discussed in the previous subsection, the measure *Power 2* reveals to what extent these high *Power 1* values are supported by generally high power at the level of the individual households. As for this second measure, we find that the variation across the different collective models is somewhat more pronounced and that, in general, the values are rather low. For example, the unitary model (i.e., for $\alpha = 0.50$) is associated with a *Power 2* value of no more than 12.64 percent: on average, about 13% of the couples do not satisfy the (unitary) empirical implications when behaving randomly. This percentage further decreases for smaller α-values. For example, when α equals 40 percent, the *Power 2* value drops to only 0.75%, which means that irrational consumption behavior is detected for an average proportion of less than 1 percent of the households.

Given our specific purpose of testing alternative behavioral models, we attribute a relatively high weight to the favorable *Power 1* results. Indeed, the construction of that measure directly complies with our practice to conclude data consistency with a behavioral model only if *all households simultaneously* pass the associated rationality tests. Still, in some instances the *Power 2* results may seem more informative. For example, generally high power estimates at the level of individual households seem recommendable when addressing recovery questions (e.g. regarding the intrahousehold allocation or the preferences of the individual household members) or forecasting issues; see, e.g., Varian (2006) for a survey of recovery and forecasting tools that are currently available in the (unitary) nonparametric approach.

From that perspective, it may be interesting to have a look at the possible causes of the relatively low *Power 2* values. One reasonable explanation for these low values lies in the fact that we have only eight observations per household: we may generally expect higher power for larger samples. Moreover, we conduct our analysis at the level of individual households. Parametric applications usually assume that at least part of the preference parameters are similar across different households, which may result in a higher power to detect alternative hypotheses. Obviously, by its very nature this parametric treatment of household heterogeneity is subject to the same risk of specification error as the parametric rationality tests themselves. In view of the particular (non-parametric testing) orientation of the current study, we believe it is recommendable to abstract from a homogeneity assumption across different households, to maximally avoid specification errors.

Another reason pertains to the assumption (in the general collective model) that we do not have any information concerning the intrahousehold allocation. Such information, for instance in the form of assignable goods (see Bourguignon, Browning and Chiappori, 2006, and Chiappori and Ekeland, 2005 and 2006), can be used to increase the power. Our general model can be adapted, to deal with such additional a priori information. For example, information on assignable goods implies additional restrictions regarding the feasible personalized prices and quantities, which in turn entails more stringent necessity and sufficiency tests for collective rationality.

Table 3.3. Sufficiency test results for couples

Model	*Number of rejections*	*Power 1*	*Power 2*
$\alpha = 0.5$	31	100	12.64
$\alpha = 0.495$	19	100	8.41
$\alpha = 0.49$	16	100	6.09
$\alpha = 0.47$	5	98.80	2.81
$\alpha = 0.45$	1	93.40	1.70
$\alpha = 0.4$	0	67.00	0.75
$\alpha = 0.3$	0	26.90	0.21
$\alpha = 0.2$	0	12.90	0.10
$\alpha = 0.01$	0	7.50	0.06
$\alpha = 0.005$	0	7.50	0.06
$\alpha = 0$ (situation-dependent dictatorship)	0	7.50	0.05

Note: Power measures are in percentages. Power 1 gives the proportion of randomly drawn data sets for which at least one household does not satisfy the tested condition. Power 2 gives the average proportion of couples that does not satisfy the tested condition across the randomly drawn data sets.

3.6 Summary and concluding remarks

This chapter presents a first empirical application of nonparametric collective rationality tests that account for public consumption and externalities within the household. Specifically, we analyzed the collective rationalization of couples that were drawn from the Russia Longitudinal Monitoring Survey (RLMS). Interestingly, the panel structure of this data set allows us to nonparametrically test the collective consumption model without relying on preference homogeneity assumptions across similar individuals.

First, we conceived an efficient procedure to test the necessity condition for the general collective consumption model, which does not put any structure on the public consumption or the within-household externalities. This procedure includes a number of efficiency enhancing mechanisms that may substantially lower the computational burden associated with the necessity tests; these operational refinements build on basic theoretical insights regarding the revealed preference relationships for individual household members. Application of these tests obtains that collective rationality cannot be rejected for the RLMS data. In addition, it shows the practical usefulness of the suggested efficiency enhancing testing strategies.

Next, we have investigated sufficiency conditions for collective rationality. We first developed a novel nonparametric framework for collective consumption models. This framework is based on the sharing rule concept, which defines the within-household distribution of the household means. Interestingly, the framework incorporates a wide range of special cases of the general collective consumption model (e.g., pertaining to observability of the intrahousehold allocation of some commodities and specific assumptions regarding the individual preferences). We then conceived operational sufficiency conditions that enable testing alternative positions regarding the specification of the household-specific sharing rules. Interestingly, these sufficient conditions for collective rationality can be conceived as direct extensions of the standard unitary rationality conditions. Specifically, the associated collective tests imply the unitary $GARP$ tests for simple transformations of the original data set.

Consistency with these sufficiency conditions means that there exists at least one definition of the collective consumption model (satisfying specific sharing rule restrictions) that rationalizes the observed behavior. Using this, our empirical investigation obtained that a multitude of collective consumption models is able to describe the couples' consumption behavior in the RLMS data. For example, we found that there certainly exists a collective rationalization of each couple within the data set under the assumption that each household member receives at least 40 percent of the total household means. By contrast, we obtained that the unitary model, which is empirically equivalent to assuming that each household member always gets the same constant share of the total means, is not able to rationalize the observed behavior.

Finally, we have analyzed the power of alternative specifications of the collective model (which correspond to different sharing rule restrictions). A first power measure captures the probability of detecting irrational behavior of at least one household in the sample. We conclude that the collective rationality tests are rather powerful at the sample level, which provides a strong support for our above empirical findings.

A second, complementary power measure captures the average or expected proportion of households of which irrational behavior is detected. The values of this measure were rather low for all model specifications (including the unitary specification). We believe this result can at least partly be explained by the availability of only eight observations per household. In this respect, it is worth noting that our (necessity and sufficiency) tests also apply to larger data sets. Such larger data sets may entail higher power at the level of individual households (captured by our second power measure). More powerful tests at the level of individual households may be especially interesting if the ultimate objective of the analysis is not so much to test data consistency with the behavioral model (as in this study) but rather to recover more detailed information regarding the intrahousehold allocation and member-specific preferences, to subsequently forecast household behavior in new situations. See for instance Varian (1982, 1983 and 2006) and Blundell, Browning and Crawford (2003, 2007), for nonparametric recovery and forecasting tools in the unitary setting.

Apart from increasing the sample size, another potentially fruitful strategy for obtaining more powerful collective rationality tests uses more stringent household-specific sharing rule restrictions (rather than a common restriction for all households, as in our study). Such restrictions can for instance be conceived on the basis of additional prior information about the intrahousehold allocation process. As we indicated, it is easy to extend the proposed testing tools for such sharing rule restrictions that vary for different households and according to the specific situation at hand.

Finally, to increase the realism of our tests , we should also develop the machinery for integrating measurement errors in our nonparametric collective tests.

Appendix

Proof of Proposition 3.3:

Suppose that we can construct the sets S_1 and S_2 of Proposition 3.3 that specify the feasible personalized quantities (see (i) and (ii)) and feasible personalized prices (see (iii)). For these feasible prices and quantities we have for all $i \in 1, \ldots, T$: if $j \in S_1$ then $(\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_j = \alpha \mathbf{p}_i' \mathbf{q}_j$ and if $j \in S_2$ then $(\widehat{\mathbf{p}}_i^1)'\widehat{\mathbf{q}}_j = (1-\alpha)\mathbf{p}_i'\mathbf{q}_j$. Therefore we obtain for each $j \in S$ that $\alpha(\mathbf{p}_j'\mathbf{q}_j) \leq (\widehat{\mathbf{p}}_j^1)'\widehat{\mathbf{q}}_j \leq (1-\alpha)(\mathbf{p}_j'\mathbf{q}_j)$ which shows that for the given feasible personalized prices and quantities, condition (i) of Definition 3.2 is met.

To prove that condition (ii) of Definition 3.2 also holds, we use Proposition 2.2, which states that there exists U^1 and U^2 that provide an *collective rationalization* of the observed set S if and only if there exist feasible personalized prices $\widehat{\mathbf{p}}_j^1, \widehat{\mathbf{p}}_j^2$ and quantities $\widehat{\mathbf{q}}_j$ such that both members satisfy the *GARP.* By the construction of S_1 and S_2, we indeed have that this latter condition holds for the given personalized prices and quantities. *Q.E.D.*

Table 3.4. Summary statistics

	Mean	*Std. dev.*
Budget shares		
Bread	0.103	0.141
Potatoes	0.010	0.054
Vegetables	0.018	0.055
Fruit	0.013	0.029
Meat	0.093	0.119
Dairy	0.047	0.063
Fat	0.025	0.049
Sugar	0.047	0.092
Eggs	0.011	0.022
Fish	0.016	0.039
Other food	0.017	0.041
Alcohol	0.014	0.041
Tobacco	0.016	0.058
Food outside the home	0.029	0.107
Clothing	0.073	0.158
Car fuel	0.054	0.123
Wood fuel	0.034	0.134
Gas fuel	0.022	0.072
Luxury goods	0.018	0.097
Services	0.191	0.222
Rent	0.146	0.170
Expenditures on nondurables	2578.30	3947.30

Note: Expenditures are in December 2003 Russian rubles per week (1 RUB = 0.03401 USD).

Chapter 4

Modeling collective rationality: nonparametric tests on experimental data

Abstract

We provide a first nonparametric test of the collective consumption model on the basis of experimental data. By using nonparametric testing tools and experimental data, we avoid the usual problems associated with parametric tests (e.g. non-verifiable parametric structure) and the use of 'real life' data sets (e.g. preference heterogeneity). In addition, our collective rationality test complements the existing nonparametric-experimental evidence on individual rationality. Our main focus is on testing the 'egoistic' collective consumption model; we perform such a (powerful) test for dyads consisting of individuals that both pass the test for individual rationality. Our test results provide strong support for the egoistic model as a tool for describing dyads' choice behavior in simple consumption decision settings, such as the one considered in our experiment. This is a useful result because the parsimonious (and powerful) egoistic model is mostly assumed in real-life applications of the collective model. Next, we find that the egoistically irrational dyads do pass our (less powerful) test for the general collective model, which accounts for consumption externalities and public consumption. This suggests that the general model can be useful even for modeling such simple decision settings, and so *a fortiori* for more complicated real-life settings.[1]

[1] This chapter is adapted from Bruyneel, Cherchye, and De Rock (2007). We are grateful to Siegfried Dewitte and André Watteyne for their helpful comments. We also thank seminar participants in Leuven and Tilburg for useful discussions.

4.1 Introduction

In this chapter, we present a *nonparametric* (revealed preference) test of the collective consumption model on the basis of *experimental* data. As argued before, the use of nonparametric tools should provide a more convincing case for the goodness of the collective model (as compared to the existing parametric evidence), essentially because it does not require debatable *a priori*'s. Moreover, the laboratory nature of experiments effectively avoids the usual preference heterogeneity and data problems. Indeed, it easily allows us to create panel data, without measurement error, for which we may reasonably assume that the preferences are static. As such, we avoid the problems with real-life data that we mentioned in Chapter 3.

In fact, it has been argued that the nonparametric testing tools are especially useful within an experimental context (see, e.g., Sippel, 1997, who focuses on individual rationality). In addition, and specific for our own study, the experimental set-up allows for obtaining information on consumption quantities for the individual group members; such information is typically not available in 'real life' data sets (e.g. household data sets usually only contain consumption quantity information at the level of the aggregate household as a whole, and do not reveal the individual members' consumption quantities). This additional information allows more powerful tests of the collective consumption model.

Our study also complements the existing nonparametric-experimental literature that focuses on the goodness of the utility maximization model for describing rational individual behavior; see, e.g., Sippel (1997), Harbaugh, Krause and Berry (2001), Andreoni and Miller (2002) and references therein. As we argued before, the collective model is the natural extension of the individual utility maximization model. Therefore, it is interesting to investigate to what extent the model indeed does succeed in describing observed group behavior, using similar nonparametric tools within an experimental context. The current study provides a first such test. More generally, it demonstrates the potential of the nonparametric analysis of experimental data for gaining insight in group decision processes; see our discussion in the concluding section.

Apart from a mere test of collective rationality as such, the second main question that we want to address pertains to the specification of the collective model itself. A simple collective consumption model, which we will refer to as the *egoistic model*, excludes public consumption and consumption externalities (also referred to as 'altruism' in the following) within the decision making group.[2] As we will show, starting from the nonparametric characterization for the general model, it is easy to define testable necessary and sufficient conditions for data consistency with the egoistic model.

Our empirical analysis mainly focuses on the egoistic model. The reason is that empirical applications of the collective model based on real-life data mostly assume this model: the parsimonious nature of the model allows for a powerful empirical analysis (e.g., in terms of recovering the preference structure of individual group members and the characteristics of the within-group bargaining process, and in terms of forecasting group behavior in new situations). In fact, this powerful nature of the egoistic model also appears in our own nonparametric tests: our tests for the egoistic model turn out to be substantially more powerful than those for the general collective model. Our laboratory test considers an unsophisticated consumption setting, which involves a very limited number of commodities and a low budget (see the experimental design in Section 4.3). The underlying argument is that, if the egoistic model is to hold in the more sophisticated settings that are usually considered in empirical applications, then it must certainly hold for this unsophisticated setting.

While our main focus is on testing the egoistic model, we also consider the general collective model. Specifically, we test consistency with this general model if observed choices are inconsistent with the egoistic model. If we subsequently conclude consistency with the general model, then we provide an interpretation in terms of consumption externalities and public consumption. In doing so, we consider the specific characteristics of the decision making groups (*in casu* dyads; e.g. partners versus friends, and differences in terms of gender composition) and the choice setting (i.e. possibility of public consumption or not). This provides a deeper insight into the usefulness of generalizing the egoistic model: if consumption externalities and public consumption can be considered relevant for the unsophisticated choice setting in our experiment, then we can expect it to be even more relevant for more sophisticated settings.

The rest of the chapter unfolds as follows. Section 4.2 introduces the nonparametric tests for the egoistic model and the Afriat efficiency

[2] This is essentially the original collective consumption model as it was presented by Chiappori (1988) (for modeling labor supply behavior).

index. Section 4.3 presents the experimental design and Section 4.4 discusses the results of our empirical analysis. Section 4.5 summarizes and contains some concluding remarks.

4.2 Collective rationality decisions: nonparametric theory

4.2.1 Collective rationality: the egoistic model

The general collective consumption model presented in Chapter 2 allows for both consumption externalities and public consumption within the dyad. In principle, any consumption commodity can be used for private consumption as well as public consumption (or combinations of both). Of course, for some commodities public consumption can be excluded *a priori* (e.g. in our own experiment this is the case for the 'private' commodities red wine, orange juice and M&Ms).

Our empirical investigation will mainly focus on a parsimonious specification of the collective consumption model, which excludes consumption externalities as well as public consumption. This obtains a model of group behavior in which the utility of each member *only* depends on her or his private consumption (i.e. $U^1(\widehat{\mathbf{q}}) = U^1(\mathfrak{q}^1)$ and $U^2(\widehat{\mathbf{q}}) = U^2(\mathfrak{q}^2)$ in Definition 2.1), whence we call it the *egoistic* model.

As mentioned before, the specific set-up of our experiment allows for recovering the personalized quantities $\mathfrak{q}^1$ and $\mathfrak{q}^2$ (if we exclude public consumption, i.e. $\mathfrak{q}^h = \mathbf{0}$). If we can observe the private consumption of each group member, we say that a commodity is 'assignable'. In the literature on collective consumption models, such assignability has proven useful for strengthening the empirical analysis. See, e.g., Browning et al., 1994, and Chiappori and Ekeland, 2006, on empirical identification of the (parametric) structure underlying the observed group behavior if (partial) assignable information is available. A similar power-enhancing effect applies to our nonparametric tests: as we will explain, the assignability information enables a powerful necessary and sufficient nonparametric test for collectively rational group behavior in terms of the egoistic model. In our opinion, this provides yet another argument pro using an experiment such as ours for testing the empirical validity of collective rationality models, because it allows us to *fully* assign each commodity under the egoistic model.

To obtain the test, we first note that excluding consumption externalities implies for each observation j that $\mathfrak{p}_j^1 = \mathbf{p}_j$ and $\mathfrak{p}_j^2 = \mathbf{0}$, so that the personalized prices $\widehat{\mathbf{p}}_j^1 = (\mathfrak{p}_j^1, \mathfrak{p}_j^2) = (\mathbf{p}_j, \mathbf{0})$ and $\widehat{\mathbf{p}}_j^2 = (\mathbf{p}_j - \mathfrak{p}_j^1, \mathbf{p}_j - \mathfrak{p}_j^2) = (\mathbf{0}, \mathbf{p}_j)$ (we can ignore public consumption (i.e. $\mathfrak{p}_j^h$) since $\mathfrak{q}_j^h = \mathbf{0}$ by construction). Hence, given that also $\mathfrak{q}_j^1$ and $\mathfrak{q}_j^2$ are fully observed, we effectively dispose of all relevant personalized price and quantity information for the egoistic model. Starting from the corresponding set of observations $\widehat{S} = \{(\mathbf{p}_j; \mathfrak{q}_j^1, \mathfrak{q}_j^2); j = 1, \ldots, T\}$ for the evaluated dyad, the nonparametric condition follows directly from Proposition 2.2:

Corollary 4.1. *Let $\widehat{S} = \{(\mathbf{p}_j; \mathfrak{q}_j^1, \mathfrak{q}_j^2); j = 1, \ldots, T\}$ be a set of observations. There exists a pair of concave and continuous utility functions U^1 and U^2 that provide an* egoistic rationalization *of S if and only if the sets $\{(\mathbf{p}_j; \mathfrak{q}_j^1);\ j = 1, \ldots, T\}$ and $\{(\mathbf{p}_j; \mathfrak{q}_j^2);\ j = 1, \ldots, T\}$ both satisfy* GARP.

Thus, testing consistency with the egoistic model is formally similar to testing consistency with the individual rationality model: for each individual member we test consistency with the *GARP* condition using the observed set $\widehat{S}$; *egoistic rationality* is obtained if both members simultaneously meet the corresponding individual rationality conditions. As such we now, in opposite to the tests in Chapters 2 and 3, have a necessary and sufficient test in terms of observable information.

Recall that in the egoistic model the group consumption process can also be characterized as a two-stage budgeting process (see Chiappori, 1988 and 1992). To recapture this alternative interpretation more formally, we represent the total dyad budget/income in observation j as y_j $(= \mathbf{p}_j' \mathbf{q}_j)$. The first stage then divides this aggregate income over the dyad members on the basis of a so-called *sharing rule*; this is a function ϕ that maps the price-income combination $(\mathbf{p}_j, y_j)$ to $\phi(\mathbf{p}_j, y_j) = (y_j^1, y_j^2)$ such that $y_j^1 + y_j^2 = y_j$, with y_j^1 and y_j^2 the income shares allocated to the members 1 and 2. In the second stage, each individual member m $(= 1, 2)$ consequently faces a maximization problem that is formally similar to the individual decision problem:

$$\max_{q^m} U^m(q^m) \text{ s.t. } \mathbf{p}_j' q^m \leq y_j^m. \tag{4.1}$$

This characterization of egoistic rationality actually provides an alternative rationale for the necessary and sufficient nonparametric condition in Corollary 4.1: given that we know the private consumption quantities $\mathfrak{q}_j^1$ and $\mathfrak{q}_j^2$, and thus also y_j^1 $(= \mathbf{p}_j'\mathfrak{q}_j^1)$ and y_j^2 $(= \mathbf{p}_j'\mathfrak{q}_j^2)$, the maximization problem (4.1) leads to the *GARP* tests for the individual group members just like in Definition 1.3.

Under egoistic rationality, the outcome of this two-stage process equals the observed consumption quantities $\mathfrak{q}_j^1$ and $\mathfrak{q}_j^2$ for the members 1 and 2. In fact, given $\widehat{S}$ we can reconstruct the sharing rule for the T observed consumption choices, namely $y_j^1 = \mathbf{p}_j'\mathfrak{q}_j^1$ and $y_j^2 = \mathbf{p}_j'\mathfrak{q}_j^2$. We will use this in our empirical (power) assessment of the egoistic rationality tests in Section 4.4.

To conclude, we note that Proposition 2.8 states that the general collective rationality condition has testable implications (i.e. can be rejected) for a dyad as soon as $T \geq 3$ and $n \geq 3$. For the egoistic model this already holds as soon as $T \geq 2$ and $n \geq 2$, since it boils down to testing *GARP* for each individual. In our application $T = 9$ and $n = 3$ *or* 4 for each evaluated dyad, so all our tests are meaningful.

4.2.2 Afriat efficiency index

To capture the degree of consistency with *GARP*, we use the 'Afriat efficiency index', which is defined as follows for each observation j:

$$\theta^j = \frac{\min\limits_{\mathbf{q}_i R \mathbf{q}_j} \mathbf{p}_j'\mathbf{q}_i}{\mathbf{p}_j'\mathbf{q}_j}; \tag{4.2}$$

the measure θ^j divides the expenditure level that is needed for obtaining consistency with *GARP* (i.e. $\min\limits_{\mathbf{q}_i R \mathbf{q}_j} \mathbf{p}_j'\mathbf{q}_i$) by the actual expenditure level (i.e. $\mathbf{p}_j'\mathbf{q}_j$). Evidently, rational (or 'efficient') behavior complies with $\theta^j = 1$. More generally, the value of θ^j captures the expenditure reduction that is required for obtaining consistency with the utility maximization problem. The corresponding Afriat efficiency index for the observed set S takes the minimum θ^j over all T choices:

$$\theta = \min\{\theta^j \mid (\mathbf{p}_j; \mathbf{q}_j) \in S\}; \tag{4.3}$$

the measure θ can be interpreted as a 'goodness-of-fit' measure in that it indicates to what extent utility maximization effectively fits the observed individual choice behavior. We refer to Varian (1990, 1993) for a detailed discussion.

Since in the egoistic model, both members have to satisfy *GARP*, we can use an analogously defined Afriat efficiency index as in (4.3) for capturing the degree of consistency with the egoistic model:

$$\begin{aligned}&\theta = \min\{\theta^j \mid (\mathbf{p}_j; \mathfrak{q}_j^1, \mathfrak{q}_j^2) \in \widehat{S}\}\\ &\text{with } \theta^j = \min\{\frac{\min_{\mathfrak{q}_i^1 R \mathfrak{q}_j^1} \mathbf{p}_j' \mathfrak{q}_i^1}{\mathbf{p}_j' \mathfrak{q}_j^1}, \frac{\min_{\mathfrak{q}_i^2 R \mathfrak{q}_j^2} \mathbf{p}_j' \mathfrak{q}_i^2}{\mathbf{p}_j' \mathfrak{q}_j^2}\}.\end{aligned} \tag{4.4}$$

As compared to (4.2), each measure θ^j now captures the expenditure reduction that is required for obtaining consistency with the *GARP* conditions in Corollary 4.1 (based on $\widehat{S}$) *for the two dyad members simultaneously.*

To end this section, we remark that the above Afriat indices in 4.2 and 4.4 are 'worse case' scenarios, given that they calculate the minimum for a given preference structure. That is, given R it computes the needed expenditure reduction for obtaining consistency with *GARP*. However, it could be that smaller expenditure reductions (than the ones suggested by the Afriat indices) also result in *GARP* consistency. More precisely, by taking into account that the expenditure reduction may alter the revealed preference structure. Therefore Varian (1990, 1993), also discusses an improved goodness-of-fit measure which tries to calculate the minimal expenditure reduction that is needed to obtain *GARP* consistency.

However, this improved goodness-of-fit measure does not change our conclusions below, given that we only use our θ in a descriptive way (see the discussion and tables in Section 4.4). More precisely, only the numbers different from 1 (i.e. when there is *GARP* inconsistency) will increase, but the main picture remains the same. Therefore, for simplicity, we opt for the above defined Afriat index.

4.3 Experimental design

Participants of our experiment were 206 undergraduate students (102 women). Ages ranged from 18 years to 29 years (mean value = 20.92; standard deviation = 1.88). As we wanted to analyze collective choice behavior, both men and women were asked to sign up for an experimental session together with either a male or a female friend or a romantic partner. This procedure enabled us to study four different types of dyads, namely, male dyads or two male friends ('*friends (m-m)*'; 26 in total), female dyads or two female friends ('*friends (f-f)*';

25 in total), mixed dyads or one male and one female friend (‘*friends (f-m)*’; 25 in total), and romantic dyads or one man and one woman who were in a romantic relationship together (‘*partners (f-m)*’; 27 in total). Participants were scheduled to come to the laboratory in groups of eight (i.e., four dyads). Each participant was assigned a seat in a partially enclosed cubicle, and worked individually for the main part of the session. Dyads were asked to engage in one experimental task together. Participants were rewarded with money and with a commodity bundle for their cooperation. Each dyad received money and a commodity bundle with a combined value of € 20. In the Appendix, we include the instructions that were handed out to the participants.

Our experiment is similar in design to the one of Harbaugh, Krause, and Berry (2001). Upon entering the laboratory, participants were given the opportunity to taste small quantities of red wine, orange juice, and M&Ms (i.e. a type of chocolate candy). They were truthfully told that they would be making consumption decisions with respect to these three commodities later on, and that we wanted them to familiarize themselves with the commodities. Participants were then presented with 9 choice problems. For (approximately) one half of the participants (i.e. in the 3-commodities condition), each choice consisted of the three commodities red wine, orange juice, and M&Ms. Each choice problem was characterized by a different price regime; the prices of the three commodities are shown in Table 4.1. We indicate that the price variation enables rejection of the general collective rationality condition in Proposition 2.2. E.g., for the given prices and income one can conceive quantity bundles that lead to a rejection of collective rationality in an analogous way as the quantity bundles in Example 2.5. This price configuration implies a high power of our rationality tests, essentially because there is no variation in income (€ 10) but a lot of variation in prices. See, e.g., Bronars (1987) and Blundell, Browning, and Crawford (2003) for a discussion in a unitary setting. Given the similarity of our collective tests, we can also apply these insights for our setting.

Table 4.1. Experimental design-prices for 9 choice problems

Choice problem	Wine	Orange juice	M&Ms
1	8	4	1
2	8	3	2
3	9	3	1
4	1	8	4
5	2	8	3
6	1	9	3
7	4	1	8
8	3	2	8
9	3	1	9

Notes: Prices are displayed in eurocents per commodity unit. A unit of red wine is 1 centiliter, a unit of orange juice is 3 centiliters, and a unit of M&Ms is 5 grams.

For each choice problem, participants were asked to indicate, according to their preferences, how much of the expenditures they wanted to pay for, and hence, how much they wanted to obtain from each commodity, given that the total budget they could allocate to the three commodities was € 10. Obviously, for this '3-commodities group' we can exclude the possibility of public consumption: the consumption of the three commodities must be private by construction. As such, the egoistic rationality test presented in the previous section effectively tests whether or not the collective decision process is characterized by consumption externalities (or altruism): in the '3-commodities condition', dyad choice data that are consistent with the general collective consumption model but not with the egoistic model can be interpreted as revealing consumption externalities.

The other half of the participants (i.e. in the '4-commodities condition') was confronted with nine almost identical decision problems (i.e. they had to state their demand for red wine, orange juice, and M&Ms, given the same relative price variations as presented in Table 4.1 and a budget of € 10). The only difference is that they had the option of receiving in cash any amount of the budget they wanted to in each decision situation; the price of this additional 'cash' commodity equals 1 for all choice problems. We note that, since the destination (public or private consumption) of the cashed amount is not defined, this '4-commodities group' has the possibility of public consumption. More generally, the fourth commodity of this 4-commodities group stands for a composite 'Hicksian good', which *in casu* captures any additional private consumption (on top of the chosen quantities of red wine, orange juice and M&Ms) and/or public consumption that is financed by the cashed amount. Thus, differences between the egoistic rationality results for the 4-commodities and the 3-commodities group may be interpreted in terms of public consumption. In fact, similar interpretations of a composite Hicksian good capturing public as well as private consumption have been used in the context of collective models of labor supply behavior (see also Blundell, Chiappori and Meghir, 2005).

Participants in both conditions were asked to make the 9 allocation decisions twice: once individually and once together with their friend or romantic partner. The order in which both sets of decisions were to be made was counterbalanced: one half of the dyads first made the decisions individually and only afterwards collectively, whereas the other half of the dyads first made the decisions collectively and only afterwards individually; changing the order in this way did not yield significantly different results in terms of (individual or collective) rationality. Table 4.11 in the Appendix presents summary information on the budget shares corresponding with the individuals' and the dyads' choices under the 9 price regimes; this expenditure information also allows for reconstructing the corresponding (mean) quantities that have been chosen under the different price regimes in Table 4.1. In case of collective decision-making, participants were asked to indicate for each of the three commodities (and also for the cashed amount, or Hicksian good, in the 4-commodities condition) which percentage of their demand was intended for each individual. This provides the personalized quantity information that we use for the egoistic rationality test (see Corollary 4.1).

The decision problems participants were faced with were supposed to mimic real-life difficulties that both individual consumers and groups often encounter when having to pick their optimal commodity bundles out of the available budget sets. To enhance the external validity of our study, participants were told that, when all experimental sessions were over (i.e., two weeks after they themselves participated at the utmost), they would actually receive one of the commodity bundles they had put together. They were also told that they would be informed through e-mail about where and when to pick it up. We picked this bundle randomly from the set of decisions that participants had made collectively (and we thus ignored the individually chosen bundles), although they were not informed of this beforehand. The knowledge that each choice ostensibly had the same chance of actually being implemented was supposed to give economic significance to otherwise merely hypothetical decisions, thus providing participants with an incentive for making choices that truly represented their preferences. More generally, we believe that the overall set-up of our experiment is such that we can have reasonable confidence in the representative nature of the choices that were made.

As making the allocation decisions required a considerable amount of calculation (multiplying prices and demand for each commodity and adding up to check whether the budget is exhausted), participants in both the 4-commodities and the 3-commodities condition were encouraged to use a calculator to check their decisions. Participants could also spend as much time as they liked on their decisions and were free to compare, reconsider, and correct previous choices. When they felt that the decisions they had made represented their actual preferences, the experimenter provided them with the instructions for the next task.

4.4 Test results

4.4.1 Individual rationality tests

We first regard test results for individual rationality. Table 4.2 reports on the individuals that violate the $GARP$ condition; it gives the number of individuals violating $GARP$ as well as some descriptive statistics for the distribution of the Afriat efficiency index values (see (4.3)) for those $GARP$ violating individuals. We find that less than 10 percent of all individuals (5 out of 104 individuals in the 4-commodities condition and 8 out of 102 in the 3-commodities condition) violate the nonparametric individual rationality condition. Still, we also find that some individuals quite severely violate the $GARP$ condition; see the minimal Afriat index values of 0.36 for the 4-commodities group and 0.48 for the 3-commodities group. But, given the small fraction of violations, these may safely be regarded as accidental outliers.

To gain some further insight into the goodness-of-fit of the individual rationality model, Table 4.3 shows the distribution of the number of $GARP$ violations (i.e. the number of observations j with $\theta^j < 1$ in (4.2)), again for the 4-commodities group and the 3-commodities group separately. Table 4.3 tells us whether the results in Table 4.2 are driven by many or by a few violations for the $GARP$ violating individuals. We find that all but one $GARP$ violating individual have no more than 4 observed consumption choices that are inconsistent with this (observation-specific) rationality condition. One individual (in the 4-commodities group) exhibits no less than 8 observed choices that are inconsistent with *GARP*. But, again, this can reasonably be considered as a casual outlier.

As a further base of comparison, we also include the distribution of violations corresponding to random behavior; see the columns '4-commodities (bootstrap)' and '3-commodities (bootstrap)' in Table 4.3. Random behavior is modeled using the bootstrap method for panel data as described by Andreoni and Miller (2002) and applied by Harbaugh, Krause and Berry (2001) within a similar experimental context.[3] The method essentially mimics random behavior for each price regime (or budget) by drawing randomly from the observed set of choices under that price regime (e.g. our experiment observes 104 choices for the 4-commodities group and 102 choices for the 3-commodities group, under 9 different price regimes). This gives information on the expected distribution of violations under random choice, while incorporating information on the participants' actual choices. All bootstrap results reported in this chapter (including those in Table 4.3) are based on Monte Carlo-type simulations that include approximately 50000 iterations.

On the basis of Table 4.3 we conclude that random behavior would yield a distribution of *GARP* violations that significantly differs from the one that is actually observed. For example, random behavior as described above would yield *GARP* consistency only in approximately 18 percent of the cases for both the 4-commodities and the 3-commodities group, as compared to no less than respectively 95 percent and 92 percent *GARP* consistencies in the observed choices. We may also interpret that the *GARP* test has a power (i.e. a probability of detecting the random behavior) of about 82 percent for both groups. In fact, random behavior entails a substantially higher probability mass for any positive number of *GARP* violations; and the relative difference

[3] This bootstrap method is similar to the randomization method proposed by Bronars (1987), which has also been used frequently in the literature. The mere difference is that 'random' choices (for each price regime) are drawn from the observed distribution whereas Bronars randomly draws from the uniform distribution (which may significantly differ from the observed distribution). We refer to Andreoni and Harbaugh (2006) for a detailed discussion of the strengths and weaknesses of alternative randomization procedures, and corresponding power measures, that have been used within a nonparametric context.

with the observed distribution generally increases for larger numbers of violations.[4]

Generally, we may conclude that the individual rationality model is strongly supported for our specific choice setting. The next question is to what extent the collective decisions, which are taken under the same 9 price regimes, are effectively consistent with collective rationality. This is discussed next.

Table 4.2. Individual rationality; GARP violations - Afriat efficiency indices; descriptive statistics

	4-commodities *(104)*	**3-commodities** *(102)*
number	5	8
maximum	0.990	0.990
3rd quartile	0.987	0.977
median	0.800	0.966
1st quartile	0.722	0.942
minimum	0.360	0.475

Notes: For each group of observations (4-commodities and 3-commodities) the total number of evaluated individuals is reported between parentheses; 'number' stands for the number of individuals violating GARP. Descriptive statistics (maximum, 3rd quartile, median, 1st quartile and minimum) pertain to the distribution of the Afriat efficiency index as defined over the subgroups of GARP violating individuals.

[4] In Subsection 4.4.3, we use an alternative bootstrap procedure for calculating the power of the sufficiency condition in Proposition 2.10. For compactness, we have not included the results of this alternative procedure for the individual rationality test. Still, it is worth indicating that this alternative procedure obtained even more favorable power results for our individual rationality test. [A similar remark holds for the egoistic rationality results in Table 4.4.]

Table 4.3. Individual rationality; distribution of GARP violations

	Percentage of group with violations			
GARP-violations	*4-com. (104)*	*3-com. (102)*	*4-com. (bootstrap)*	*3-com. (bootstrap)*
0	95.2	92.2	18.2	18.1
1	1.0	2.9	5.5	2.4
2	1.0	3.9	17.2	18.6
3	1.0	0.0	13.1	11.7
4	1.0	1.0	13.3	13.5
5	0.0	0.0	11.9	12.5
6	0.0	0.0	9.5	10.4
7	0.0	0.0	6.3	7.0
8	1.0	0.0	3.6	4.1
9	0.0	0.0	1.4	1.6

Notes: For each group of observations (4-commodities and 3-commodities) the total number of evaluated individuals is reported between parentheses. The column 'GARP violations' stands for the number of GARP violations (ranging from minimally 0 to maximally 9). For each group (4-commodities and 3-commodities), the table reports the percentage of (observed and random/'bootstrap') choices corresponding to different numbers of violations.

4.4.2 Egoistic rationality tests

Individually rational behavior is a necessary condition for collectively rational behavior. Therefore, we investigate data consistency with the collective consumption model for those dyads of which both members act consistent with the individual rationality test. This obtains 47 dyads (94 individuals) for the 4-commodities group and 43 dyads (86 individuals) for the 3-commodities group.[5]

As argued before, our main focus is on the egoistic model. The fact that our experiment fully recovers the personalized quantities allows for testing a necessary and sufficient condition for egoistic rationality, which essentially consists of two $GARP$ tests per dyad, i.e. one for each individual member (see Corollary 4.1). Consistency of the collective

[5] We note that excluding the couples with 'irrational' individuals in this way does not affect our main qualitative conclusions. Still, because individual rationality is a prerequisite for collective rationality, we find it logically consistent to focus our discussion on couples with rational singles. A slightly different approach is followed in Subsection 4.4.3, when we regard alternative efficiency criteria that allow for (small) deviations from 'exactly' rational behavior (as captured by the Afriat efficiency index). See our discussion of Table 4.8.

decisions with the 'rudimentary' egoistic model should not be counter-intuitive for the unsophisticated decision setting under study: it may well be that none of the commodities taken up in the experiment is associated with consumption externalities; and (for the 4-commodities group) the cashed amount (or Hicksian good), which does allow for public consumption, should of course not necessarily be used for such public consumption. At least, we can argue that, if the egoistic model is to hold in more sophisticated settings (as is mostly assumed in empirical applications), then it must certainly hold in this setting.

Table 4.4 has a similar interpretation as Table 4.2, but now pertains to egoistic rationality; it gives information on violations of the corresponding *GARP* conditions at the level of the individual dyads' members. We find that less than ten percent of all observations (9 out of 94 individual members for the 4-commodities case and 8 out of 86 members for the 3-commodities case) is inconsistent with the egoistic rationality condition. Of course, *stricto sensu* consistency with the egoistic model (see Corollary 4.1) requires that both individual members simultaneously meet the corresponding *GARP* condition. When using that criterion we find that 39 out of 47 dyads (83 percent) in the 4-commodities group and 37 out of 43 dyads (86 percent) in the 3-commodities group behave egoistically rational. See also our following discussion of Table 4.8.
The other descriptive statistics are closely similar to those in Table 4.2 for individual rationality. Table 4.5 gives the corresponding distribution of the number of (egoistic rationality) *GARP* violations. Once more, we find strong support for the (*in casu* egoistic) collective rationality model. For example, no individual member exhibits more than three *GARP* violations.

Table 4.5 also gives the distribution of the *GARP* violations under random behavior. To obtain these results, we use a similar randomization procedure as for computing the results in Table 4.3. But in this case we exploit the two-step budgeting interpretation of the egoistic model; see our discussion leading up to the decision problem (4.1) for each individual dyad member. This two-step structure underlies our bootstrap procedure for constructing random choices, as follows: for each price regime, we randomly select the income share for each individual member from the observed distribution; and, subsequently, we randomly select the budget allocation (i.e. the relative budget shares), again drawing from the observed distribution corresponding to the given price regime. The resulting bootstrap distributions (for the 4-commodities and the 3-commodities group) are based on 50000 such random choices for the 9 price regimes. We find that these distributions are very different from the ones that are observed. The corresponding power estimates amount to about 76 percent for the 4-commodities group and 72 percent for the 3-commodities group. While slightly below the power of the individual rationality test (see Table 4.3), the distribution of $GARP$ violations for the observed choice behavior (including the observed proportion of $GARP$ inconsistencies) is sufficiently different from the one corresponding to random behavior to safely conclude strong support of the egoistic rationality model.

As a general conclusion we can state that our results provide convincing support for the collective rationality model, which here takes the form of the parsimonious (and therefore powerful) egoistic rationality model. The fact that this 'rudimentary' version of the collective decision model adequately describes most of the observed behavior in our experiment seems intuitively reasonable given the simple choice setting at hand. Still, it also seems interesting to consider in somewhat more detail the about 10 percent observations that are inconsistent with this egoistic model, in particular because the results in Tables 4.4 and 4.5 pertain to dyads for which both individuals act consistent with the individual rationality condition. Therefore, in the next section we investigate data consistency with the general collective consumption model. That is, we explore whether it is possible to 'rationalize' the observed egoistic irrationality in terms of consumption externalities and public consumption.

Table 4.4. Egoistic rationality; GARP violations - Afriat efficiency indices; descriptive statistics

	4-commodities	**3-commodities**
	(94)	*(86)*
number	9	8
maximum	0.991	0.999
3rd quartile	0.986	0.997
median	0.944	0.980
1st quartile	0.550	0.958
minimum	0.521	0.925

Notes: For each group of observations (4-commodities and 3-commodities) the total number of evaluated individual members is reported between parentheses; 'number' stands for the number of individual members violating GARP corresponding with egoistic rationality. Descriptive statistics (maximum, 3rd quartile, median, 1st quartile and minimum) pertain to the distribution of the Afriat efficiency index as defined over the subgroups of GARP violating individual members.

Table 4.5. Egoistic rationality; distribution of GARP violations

	Percentage of group with violations			
GARP-violations	*4-com. (94)*	*3-com. (86)*	*4-com. (bootstrap)*	*3-com. (bootstrap)*
0	90.4	90.7	24.1	21.9
1	1.1	0.0	1.7	0.4
2	6.4	9.3	18.5	17.7
3	2.1	0.0	11.0	10.4
4	0.0	0.0	12.8	12.7
5	0.0	0.0	11.3	11.6
6	0.0	0.0	9.3	10.5
7	0.0	0.0	6.4	7.7
8	0.0	0.0	3.6	5.0
9	0.0	0.0	1.4	2.2

Notes: For each group of observations (4-commodities and 3-commodities) the total number of evaluated individual members is reported between parentheses. The column 'GARP violations' stands for the number of GARP violations corresponding with the egoistic rationality model (ranging from minimally 0 to maximally 9). For each group (4-commodities and 3-commodities), the table reports the percentage of (observed and bootstrap) choices corresponding to different numbers of violations.

4.4.3 'Rationalizing' egoistic irrationality

When testing the general rationality condition in Proposition 2.2, we conclude that all 'egoistically irrational' dyads act fully consistent with the general collective model. Specifically, we find that all but one of the dyads are consistent with the sufficient condition in Proposition 2.10. The Appendix proves consistency with the collective rationality condition for the remaining one dyad (from the 4-commodities group) that is inconsistent with the sufficiency condition: it presents a specification of personalized prices and quantities that makes the dyad's observed behavior obey the necessary and sufficient condition in Proposition 2.2.

This positive result for the general model suggests that consumption externalities and/or public consumption can 'rationalize' the observed egoistic irrationality. To further explore this possibility, we will regard in more detail the egoistic rationality violations of the 4-commodities group and of the 3-commodities group. In addition, recalling our discussion of the experimental design in Section 4.3, we use the distinction between four different types of dyads, namely *friends (m-m)*, *friends (f-f)*, *friends (f-m)* and *partners (f-m)*.

We first regard consumption externalities (or altruism). As explained before, we can exclude public consumption for the 3-commodities group, which implies that the egoistic rationality test effectively boils down to testing for the presence of such externalities. Given this, we regard the lower panel of Table 4.6, which has a similar structure as Table 4.4 but now presents the results for the four dyad types under investigation. Table 4.7 similarly decomposes the aggregate (4-commodities and 3-commodities) results in Table 4.5. According to the results in these tables, 'altruistic' decision makers (who account for consumption externalities in their consumption decisions) seem to be situated in dyads containing two friends with at least one female member (i.e. the types *friends (f-m)* and *friends (f-f)*). Interestingly, this result falls in line with existing evidence that females are more altruistic towards friends and other close individuals than males (see, e.g., the meta-analysis of Eagley and Crowley, 1986; Croson and Gneezy, 2005, provide a recent review of studies on gender differences in preferences).[6] In that interpretation, the observation that all dyads of the category *partners (f-m)*

[6] In this respect, it is also interesting to note that our results on the sharing rule seem to comply with existing evidence that females propose equal split more than males (see Croson and Gneezy, 2005, for a survey of results). For example, an equal split of the income under all 9 price regimes, when using the specification of personalized prices that applies under egoistic rationality, is used by no less than 37.5 percent of all dyads in the category *friends (f-f)* and 33.33 percent of all dyads in the category *friends (f-m),* as opposed to only 13.0 percent of all dyads in the category *friends (m-m)* and 12.0 percent of all dyads in the category *partners (f-m).* In our opinion, a detailed investigation of these sharing

in the 3-commodities group are consistent with egoistic rationality may seem paradoxical. Still, this last result might be explained by the fact that for these dyads the consumption decisions in our experiment (for a joint budget of € 10) account for only a small proportion of their collective decisions; and thus consumption externalities (altruism) may well affect other ('more important') collective choices not taken up in this experiment. Of course, the evidence in Table 4.6 is far too weak for drawing robust conclusions; see the small number of violations of the egoistic rationality conditions, and the high Afriat efficiency index values (with overall minimum = 0.925). Although these results should be interpreted as indicative rather than conclusive, they do suggest the nonparametric collective rationality tests proposed in this chapter as potentially useful tools for investigating this type of questions; specially targeted experiments may help to further investigate the observed patterns in greater detail.

Let us then consider the possibility of public consumption. As indicated in Section 4.3, public consumption is possible for the 4-commodities group (through the cashed amount, which is interpreted as a Hicksian good) and not for the 3-commodities group. Therefore, comparing the violations of egoistic rationality for the 4-commodities group with those for the 3-commodities group may reveal whether or not public consumption is relevant within our choice setting. Again, we make the distinction between the four dyad types. Comparison of the upper and lower panels of Tables 4.6 and 4.7 indeed seems to confirm that violation of the egoistic rationality model may be rationalized through public consumption, thus suggesting that such public consumption is a relevant dimension of collective decision making even for unsophisticated choice settings. Specifically, recall that our results for the 3-commodities group suggest the absence of consumption externalities for the dyad types *friends (m-m)* and *partners (f-m)* in the simple choice setting of our experiment. Under that maintained assumption, public consumption may explain the observed violations of collective rationality for the 4-commodities group. As for the other categories *friends (f-m)* and *friends (f-f)*, because we did observe violations of egoistic rationality in the 3-commodities condition, we cannot distinguish between consumption externalities and public consumption in the 4-commodities condition. Still, we do find that the violations of ego-

rule mechanics may constitute yet another interesting research avenue in which the nonparametric collective rationality testing tools can be instrumental.

istic rationality are more severe when public consumption is possible; see in particular the changes of the descriptive statistics of the Afriat efficiency index for these two dyad types. Once more, we should stress that the current set-up only allows for tentative conclusions.

So far we have considered egoistic rationality tests at the level of the individual dyad members. Of course, collective rationality and thus also egoistic rationality should actually be considered at the dyad level: a dyad behaves egoistically rational only if both individual members *simultaneously* meet the corresponding $GARP$ condition. Our final Table 4.8 reports such dyad level egoistic rationality results. Specifically, it gives the number of dyad observations and the percentage of so-called 'egoistically rational' dyads (again subdivided by type) for alternative criteria expressed in terms of the Afriat efficiency index (4.4): each $X\%$ efficiency criterion ($X = 100$, 99 or 95) states that a dyad is $X\%$ egoistically rational if the corresponding Afriat efficiency index is at least $X\%$ (e.g. the earlier results in Tables 4.6 and 4.7 comply with the 100% efficiency criterion). To be exact, each *X%* efficiency criterion only considers dyad observations for which each *individual* dyad member passes the similarly defined *X%* efficiency criterion defined in terms of the Afriat efficiency index (4.3) *for individuals.* This makes that the number of dyad observations increases for lower X. Varian (1990) suggested a similar efficiency criterion for individual rationality, and he proposed the 95% cut-off level. We apply the idea to collective rationality, and additionally consider the 100% and 99% cut-off levels.

The results in Table 4.8 support the same (tentative) conclusions as before. From the 3-commodities results, we may derive that altruism helps in explaining violations of egoistic rationality for dyads of the types *friends (f-f)* and *friends (f-m)*. For example, we find that only 63.6 (75, 92.3) percent dyads of the type *friends (f-f)* are consistent with the 100% (99%, 95%) egoistic rationality criterion. A similar, albeit less strongly marked, result also applies to the category *friends (f-m)*. In addition, comparison of the 3-commodities results and the 4-commodities results indicates that public consumption can rationalize egoistically irrational behavior of, most notably, dyads of the types *friends (m-m)* and *partners (f-m)*. For example, while 100 percent dyads of the type *friends (m-m)* are consistent with egoistic rationality in the 3-commodities condition (for all three efficiency criteria), only 78.6 (85.7, 92.9) percent is consistent with the 100% (99%, 95%) egoistic rationality criterion in the 4-commodities condition. Similar differences, albeit to a somewhat lesser extent, hold for the category *partners (f-m)*.

We may conclude that our findings at least suggest that both consumption externalities and public consumption are relevant for modeling collective decisions, even in simple choice settings such as the one of our experiment. Obviously, we may expect these features to become even more important in real-life applications that are generally characterized by more complicated collective decisions. In turn, this suggests a general usefulness of the general collective consumption model.

As a final exercise, we compute the power of the sufficiency condition in Proposition 2.10 for the general collective consumption model; this power estimate can be interpreted as an upper bound for the power of the necessary and sufficient condition in Proposition 2.2. We first consider a similar randomization procedure as before: for each price regime we randomly draw consumption choices from the set of observed dyads' choices (47 for the 4-commodities group and 43 for the 3-commodities group). The resulting power estimate equals 4.6 percent for the 4-commodities case and 5.0 percent for the 3-commodities case. While these figures are effectively above the percentage of violations that is actually observed (namely 1/47 percent for the 4-commodities case and zero percent for the 3-commodities case), they are far below the power estimates that apply for the individual and egoistic rationality tests.

At this point, it is worth indicating that the randomization procedure that is used for computing the power may be subject to criticism; it puts a lot of prior structure on the so-called 'random' choices by conditioning their selection on the price regime. An alternative randomization procedure does not consider the specific price regime, but considers the *full* set of all actually observed budget allocations (i.e. relative budget shares) as potentially 'random' choices for *every* price regime. For our application, this implies for each price regime 9×47 possible choices for the 4-commodities case and 9×43 possible choices for the 3-commodities case. This alternative procedure obtains a power estimate for the sufficiency test that equals 15.3 percent for the 4-commodities group and 13.2 percent for the 3-commodities group. While these estimates are obviously more favorable for the general collective rationality test, they are again rather low. These results signal a general need for methods that increase the power of the nonparametric tests for the general collective consumption model, of which the usefulness has been argued before. We discuss some possible avenues in the following concluding section.

Table 4.6. Egoistic rationality; GARP violations - Afriat efficiency indices; descriptive statistics per type

4-commodities	*partners (f-m)*	*friends (f-m)*	*friends (f-f)*	*friends (m-m)*
	(26)	*(22)*	*(18)*	*(28)*
number	1	2	3	3
maximum	0.944	0.984	0.990	0.991
3rd quartile	0.944	0.868	0.770	0.989
median	0.944	0.752	0.550	0.986
1st quartile	0.944	0.636	0.550	0.918
minimum	0.944	0.521	0.550	0.851
3-commodities	*partners (f-m)*	*friends (f-m)*	*friends (f-f)*	*friends (m-m)*
	(22)	*(24)*	*(22)*	*(18)*
number	0	3	5	0
maximum	-	0.996	0.999	-
3rd quartile	-	0.961	0.998	-
median	-	0.925	0.980	-
1st quartile	-	0.925	0.980	-
minimum	-	0.925	0.969	-

Notes: For each group of observations (4-commodities and 3-commodities; four dyad types) the total number of evaluated individual members is reported between parentheses; 'number' stands for the number of individual members violating GARP corresponding with egoistic rationality. Descriptive statistics (maximum, 3rd quartile, median, 1st quartile and minimum) pertain to the distribution of the Afriat efficiency index as defined over the subgroups of GARP violating individual members.

Table 4.7. Egoistic rationality; distribution of GARP violations per type

	Percentage of group with violations (4-commodities)			
GARP-violations	*partners (f-m) (26)*	*friends (f-m) (22)*	*friends (f-f) (18)*	*friends (m-m) (28)*
0	96.2	90.9	83.3	89.3
1	0.0	0.0	0.0	3.6
2	3.8	9.1	5.6	7.1
3	0.0	0.0	11.1	0.0
4	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0
	Percentage of group with violations (3-commodities)			
GARP-violations	*partners (f-m) (22)*	*friends (f-m) (24)*	*friends (f-f) (22)*	*friends (m-m) (18)*
0	100.0	87.5	77.3	100.0
1	0.0	0.0	0.0	0.0
2	0.0	12.5	22.7	0.0
3	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0

Notes: For each group of observations (4-commodities and 3-commodities; four dyad types) the total number of evaluated individual members is reported between parentheses. The column 'GARP violations' stands for the number of GARP violations corresponding with the egoistic rationality model (ranging from minimally 0 to maximally 9). For each group, the table reports the percentage of observed choices corresponding to different numbers of violations.

Table 4.8. Egoistic rationality at the dyad level; alternative efficiency criteria; per type

	100% eff. crit.			**99% eff. crit.**			**95% eff. crit.**		
	all	*4-com.*	*3-com.*	all	*4-com.*	*3-com.*	*all*	*4-com.*	*3-com.*
all types									
number	90	47	43	92	48	44	98	49	49
% pass test	84.4	83.0	86.0	89.1	87.5	90.9	93.9	91.8	95.9
partners (f-m)									
number	24	13	11	25	14	11	26	15	11
% pass test	95.8	92.3	100.0	96.0	92.9	100.0	96.2	93.3	100.0
friends (f-m)									
number	23	11	12	23	11	12	24	11	13
% pass test	82.6	81.8	83.3	87.0	81.8	91.7	91.7	90.9	92.3
friends (f-f)									
number	20	9	11	21	9	12	22	9	13
% pass test	70.0	77.8	63.6	81.0	88.9	75.0	90.9	88.9	92.3
friends (m-m)									
number	23	14	9	23	14	9	26	14	12
% pass test	87.0	78.6	100.0	91.3	85.7	100.0	96.2	92.9	100.0

Notes: For each group of observations (4-commodities and 3-commodities; four dyad types) the table reports the efficiency results according to alternative X% efficiency criteria (X = 100, 99, 95). For a given X the row 'number' gives the number of dyad observations with an Afriat efficiency index (4.3) of at least X% for each individual member; and the row '% pass test' gives the percentage of such dyad observations with an Afriat efficiency index (4.4) of at least X%.

4.5 Summary and concluding remarks

We have provided a first nonparametric-experimental test of the collective consumption model. First, focusing on dyads consisting of individuals that both pass the test for individual rationality, we have performed a test for collective rationality in terms of the egoistic model. Our test results provide strong support for egoistically rational dyad behavior in simple consumption decision settings, such as the one considered in our experiment: a large proportion of dyads (i.e. above 90 percent) is effectively consistent with the (powerful) egoistic rationality conditions. This is a useful result, as the parsimonious egoistic model is mostly assumed in real-life empirical applications, which are usually characterized by more sophisticated decision settings; a minimal test for the validity of that assumption is that the model holds in simple consumption settings. Second, we find that observed dyad behavior that is inconsistent with the egoistic model turns out to be consistent with the general collective consumption model, which incorporates the possibility of consumption externalities and public consumption. This result suggest that this general model can be useful even for modeling such simple decision settings, and so *a fortiori* for more complicated real-life settings.

At a more general level, we believe that this first test demonstrates that the nonparametric (collective consumption) analysis of experimental data can be particularly useful for gaining insight in the mechanics of group decision making under alternative choice conditions. For example, our own empirical application suggests that in particular female friends seem to behave altruistically, which corresponds to consumption externalities, and that some dyads effectively seem to use (even fairly small) amounts of cash for public consumption (captured by a composite Hicksian good in our set-up). Still, although these rationalization arguments have intuitive appeal, they can at best be interpreted as tentative, mainly because of the small number of egoistically irrational dyads in our experiment. Follow-up research can focus on experimental choice settings that are specially designed for studying the specificities that cause consumption externalities (or altruism) and public consumption; this complements the existing literature that focuses on altruism in individual decision making (see, e.g., Andreoni and Miller, 2002).

Alternative research questions can relate to other specific features of the collective decision process. For example, follow-up research that uses experimental data may concentrate on the bargaining idea (including the determinants of the bargaining power) that underlies the collective consumption model. Or, whereas this first study restricted attention to dyads, future research may investigate group decisions that involve more than two decision makers. In such settings, one may e.g. be interested in the number of decision makers that are effectively involved in the group decision process. That is, how many decision makers have to be accounted for in order to make the observed group behavior consistent with collective rationality? In a family context, a closely related research question is whether and to what extent children bear on the bargaining process or do have actual bargaining power within households. From a closely related perspective, one can study the group decision behavior of young children, including the nature of the underlying preferences (egoistic or altruistic); this would complement the results of Harbaugh, Krause and Berry (2001) on the individual rationality of young children. Once more, specially targeted experiments that use the nonparametric collective rationality tests may enhance our understanding of these issues.

A final important conclusion of our study pertains to the specification of the rationality tests themselves. In particular, our comparison of the general collective rationality results with the egoistic rationality results shows that knowledge of personalized quantities and prices (which in our case were fully observed under egoistic rationality) may dramatically increase the power of the tests. From a practical perspective, more powerful tests obviously imply more powerful recovery and forecasting results. [See Varian (1982, 2006) for surveys of nonparametric recovery and forecasting tools that build on the *GARP* test for individual rationality; these tools could be adapted to the collective consumption context.] This pleads for developing collective consumption data sets that incorporate such personalized quantity and price information; such detailed data sets seem especially valuable given the good fit of the collective consumption model.

Two final remarks are in order with respect to increasing the power of the nonparametric tests in practical applications. First, it is clear that, if we knew the individual members' orderings of the (collectively chosen) consumption bundles, then we could design more powerful tests of collectively rational behavior. Such tests would have a formally similar structure as the ones in Chapter 5 that apply in a production setting (where the observed outputs reveal the ordering information). Within experimental set-ups it is actually possible to directly ask the participants for the ordering information; this suggests an interesting exercise for follow-up research. Second, the power of the nonparametric tests may be increased by extending the existing tests to include the 'sequential maximum power path' idea of Blundell, Browning and Crawford (2003,2007), who originally focused on the *GARP* condition for individual rationality. Such an extension could render the nonparametric toolkit for the collective model, which effectively seems to provide an adequate description of the observed group decision behavior, particularly useful for addressing real-life research questions (such as the prediction of group behavior in new situations and/or welfare analysis).

Appendix

Additional information: instructions for experiment

For completeness, given that this chapter uses experimental data, we also add the instructions that were given to the participants of the experiment and which are not explicitly taken up in the main text:

Introduction and product-tasting

In a moment, you will be asked to make a series of choices regarding the three products standing on the desk in front of you: wine, orange juice, and M&Ms.
In order to enable you to make these choices in an informed way, you have the opportunity to taste these three products (wine, orange juice, and M&Ms) right now. You can consume everything if you want to.
You will be asked to make the choices partly on your own and partly together with the person who accompanied you to the lab.
It is in your own and in your joint interest to make these choices as truthfully as possible, that is, to make the same choices you would in real-life. This is because one of your choices will be randomly selected for you to take home.
Because of practical reasons, you will not receive your choice right away. You will however be invited to pick up your choice afterwards. Each choice will consist of a product package (of wine, orange juice, and M&Ms) that is worth € 10.

Decision-making

In the following, you are asked to make a series of choices. Each time, relative prices of three different products (wine, orange juice, and M&Ms) are given. It is up to you to decide how much you are willing to spend on each product (and, added in the 4-commodities condition: and how much you possibly would like to save), given these relative price variations. Each time, your budget amounts to €10 or 1000 eurocents. Please make each choice as truthfully as possible, that is, as you would in real life, as one of your choices will be randomly selected for you to take home.
For each choice that you make as a dyad, you have to indicate how you divide the chosen products over the individuals in the dyad.
If you want to, you can use the computer's calculator. You are also free to use the back of the questionnaire for scrap paper if you want to.

Collective rationalization in terms of Proposition 2.2

We provide a collective rationalization in terms of the nonparametric conditions in Proposition 2.2 for the one dyad observation that does not meet the sufficiency condition in Proposition 2.10. Table 4.9 reports the observed aggregate (dyad level) quantities and the corresponding (member level) personalized quantities for this dyad. Combining the aggregate quantity information with the price information in Table 4.1 (and using that the price of cash equals 1), it can be verified that this dyad does not meet the sufficiency condition. Specifically, we have for the three choice observations 2, 3 and 5 that for each pair $i, j \in \{2, 3, 5\} : \mathbf{p}_i'\mathbf{q}_i > \mathbf{p}_i'\mathbf{q}_j$ and $\mathbf{p}_j'\mathbf{q}_j > \mathbf{p}_j'\mathbf{q}_i$. In such a case there does not exist a partitioning of the observed set S (of aggregate quantities and prices) that distributes these three observations over two subsets S_1 and S_2 so that each individual subset meets the corresponding *GARP* condition.

Still, the dyad does meet the necessary and sufficient condition in Proposition 2.2. For example, Table 4.10 gives a feasible specification of the personalized prices for which such consistency can be verified: assuming that the cashed amounts are pooled to spend on a publicly consumed good (i.e. the Hicksian good is a pure 'public good', with an aggregate price of unity), both dyad members simultaneously meet the corresponding *GARP* conditions in terms of the personalized quantities (for the goods wine, orange juice and M&Ms) in Table 4.9 and the corresponding personalized prices in Table 4.10. Of course, such a data rationalizing specification of the personalized prices should in general not be unique; alternative specifications can obtain the same consistency result (e.g.: our interpretation of cashed amounts in terms of public consumption is not necessary for establishing the consistency).
collectively rationalizing personalized prices for the 9 choice problems

Table 4.9. Aggregate and personalized quantities for the 9 choice problems

Choice problem	**Wine**	**Orange juice**	**M&Ms**	**Public good**
		Aggregate quantities		
1	75	50	200	0
2	75	0	100	200
3	75	50	75	100
4	450	0	0	550
5	0	50	100	300
6	450	0	100	250
7	0	400	0	600
8	225	112	0	100
9	100	600	0	100
		Personalized quantities member 1		
1	37.5	25	100	0
2	37.5	0	50	100
3	37.5	25	37.5	50
4	225	0	0	275
5	0	25	50	150
6	225	0	50	125
7	0	200	0	300
8	112.5	56	0	0
9	50	300	0	50
		Personalized quantities member 2		
1	37.5	25	100	0
2	37.5	0	50	100
3	37.5	25	37.5	50
4	225	0	0	275
5	0	25	50	150
6	225	0	50	125
7	0	200	0	300
8	112.5	56	0	100
9	50	300	0	50

Notes: Consumption quantities are expressed in terms of the same commodity units as the prices in Table 4.1: a unit of red wine is 1 centiliter, a unit of orange juice is 3 centiliters, and a unit of M&Ms is 5 grams.

Table 4.10. Collectively rationalizing personalized prices for the 9 choice problems

Choice problem	**Wine**	**Orange juice**	**M&Ms**	**Wine**	**Orange juice**	**M&Ms**	**Public good**
	Pers. prices member 1 *own (= member 1) cons.*			*Pers. prices* member 1 *other (= member 2) cons.*			*Pers. price* member 1
1	8	4	1	8	4	1	0.5
2	8	3	2	8	3	2	0.5
3	9	3	1	9	3	1	0.5
4	1	8	4	1	8	4	0.5
5	0	0	0	0	0	0	0.5
6	1	9	3	1	9	3	0.5
7	4	1	8	4	1	8	0.5
8	3	2	8	3	2	8	0
9	3	1	9	3	1	9	0.5
	Pers. prices member 2 *own (= member 2) cons.*			*Pers. prices* member 2 *other (= member 1) cons.*			*Pers. price* member 2
1	0	0	0	0	0	0	0.5
2	0	0	0	0	0	0	0.5
3	0	0	0	0	0	0	0.5
4	0	0	0	0	0	0	0.5
5	2	8	3	2	8	3	0.5
6	0	0	0	0	0	0	0.5
7	0	0	0	0	0	0	0.5
8	0	0	0	0	0	0	1
9	0	0	0	0	0	0	0.5

Table 4.11. Experimental results - budget shares for the 9 choice problems

Choice problem		Wine	Orange juice	M&Ms	Hicksian	Wine	Orange juice	M&Ms	Hicksian
		4-com. - 104 ind.				*4-com. - 52 dyads*			
1	*mean*	0.186	0.209	0.208	0.395	0.226	0.228	0.232	0.315
	st. dev.	0.218	0.181	0.230	0.285	0.243	0.197	0.239	0.267
2	*mean*	0.182	0.247	0.184	0.387	0.215	0.222	0.200	0.347
	st. dev.	0.216	0.205	0.195	0.277	0.246	0.199	0.200	0.265
3	*mean*	0.164	0.243	0.219	0.374	0.174	0.243	0.263	0.317
	st. dev.	0.213	0.204	0.239	0.284	0.231	0.218	0.262	0.278
4	*mean*	0.345	0.138	0.123	0.393	0.373	0.132	0.153	0.337
	st. dev.	0.315	0.188	0.179	0.286	0.332	0.182	0.207	0.273
5	*mean*	0.325	0.153	0.133	0.388	0.318	0.146	0.194	0.342
	st. dev.	0.273	0.200	0.174	0.270	0.288	0.187	0.224	0.270
6	*mean*	0.342	0.142	0.143	0.373	0.372	0.111	0.200	0.318
	st. dev.	0.305	0.211	0.196	0.289	0.334	0.172	0.246	0.279
7	*mean*	0.214	0.323	0.109	0.347	0.226	0.299	0.133	0.340
	st. dev.	0.230	0.290	0.180	0.277	0.254	0.272	0.213	0.294
8	*mean*	0.253	0.271	0.109	0.366	0.280	0.251	0.131	0.330
	st. dev.	0.234	0.234	0.178	0.271	0.242	0.186	0.191	0.274
9	*mean*	0.235	0.307	0.107	0.354	0.261	0.290	0.132	0.319
	st. dev.	0.237	0.269	0.191	0.286	0.277	0.264	0.230	0.287
		3-com. - 102 ind.				*3-com. - 51 dyads*			
1	*mean*	0.310	0.361	0.329		0.261	0.331	0.407	
	st. dev.	0.296	0.248	0.297		0.291	0.226	0.282	
2	*mean*	0.299	0.382	0.320		0.261	0.369	0.370	
	st. dev.	0.292	0.260	0.292		0.303	0.245	0.283	
3	*mean*	0.261	0.381	0.355		0.211	0.369	0.420	
	st. dev.	0.289	0.276	0.305		0.271	0.247	0.299	
4	*mean*	0.457	0.251	0.293		0.432	0.232	0.333	
	st. dev.	0.354	0.255	0.301		0.360	0.253	0.315	
5	*mean*	0.421	0.238	0.338		0.392	0.234	0.376	
	st. dev.	0.323	0.234	0.293		0.329	0.233	0.299	
6	*mean*	0.454	0.217	0.323		0.440	0.189	0.367	
	st. dev.	0.346	0.250	0.308		0.367	0.241	0.328	
7	*mean*	0.307	0.443	0.240		0.289	0.435	0.276	
	st. dev.	0.315	0.321	0.301		0.303	0.304	0.315	
8	*mean*	0.344	0.434	0.227		0.316	0.396	0.289	
	st. dev.	0.303	0.294	0.292		0.308	0.261	0.305	
9	*mean*	0.340	0.429	0.231		0.329	0.411	0.262	
	st. dev.	0.318	0.321	0.316		0.332	0.332	0.342	

Notes: For each choice problem and each group of observations (4-commodities and 3-commodities), the table reports the mean budget shares ('mean') over all participants (individuals and dyads), together with the corresponding standard deviations ('st.dev.').

Chapter 5

Collective approach to firm production: cost efficiency analysis of multi-output firms

Abstract

In designing a production model for firms that generate multiple outputs, we take as a starting point that such multi-output production refers to economies of scope, which in turn originate from joint input use and input externalities. We provide a nonparametric characterization of cost efficient behavior under these conditions, and subsequently institute necessary and sufficient conditions for data consistency with such efficient behavior that only include observed firm demand and supply data. We illustrate our methodology by examining the cost efficiency of research programs in Economics and Business Management faculties of Dutch universities. This application shows that the proposed methodology may entail robust conclusions regarding cost efficiency differences between universities within specific specialization areas, even when using shadow prices to evaluate the different inputs.[1]

[1] This chapter is adapted from Cherchye, De Rock and Vermeulen (2007b). We are grateful to two anonymous referees for insightful comments and suggestions. We also thank seminar participants at the European Workshop on Efficiency and Productivity Analysis 2007 in Lille for useful discussions.

5.1 Introduction

In this chapter we exploit the conceptual analogy between the collective model of multi-person household consumption (with public consumption and consumption externalities) and the 'scope economies'-based model of multi-output firm production (with joint inputs and production externalities). Deviating from the mainstream literature (see Cooper, Seiford and Tone, 2000, and Fried, Lovell and Schmidt, 2007, for surveys), we start from the position that such multi-output production refers to economies of scope, which in turn originate from joint input use and production externalities. Economies of scope may loosely be defined as situations where the average total cost of production decreases as a result of increasing the number of different goods produced (see, e.g., Baumol, Panzar and Willig, 1982). Given this, we stress that we do not want to present a methodology for investigating the extent to which economies of scope are actually present. Rather, as we explain below, we present a nonparametric toolkit for analyzing cost efficient production behavior that exploits a number of specific features related to scope economies. This is an important difference between our approach and most other contributions on scope economies in the production literature, which indeed essentially aim at recovering whether and to what extent the production technology under study is characterized by economies of scope. See, e.g., Kim et al. (2005) and references therein.

As before, a first specificity of our approach is that it is embedded in a nonparametric methodology. Also in the production setting this approach has some well-known advantages when compared to a parametric approach.[2] For example, it does not rely on a functional specification of a firm's production technology, which again is typically non-testable. Further, it deals in a very natural way with the widely observed simultaneous occurrence of multiple inputs and outputs. Finally, it easily accounts for the possibility that input-output combinations do not necessarily have to lie on the production frontier: production behavior can be analyzed while allowing for observed inefficiencies.[3]

[2] See, e.g., Varian (1984), Färe, Grosskopf and Lovell (1994) and Cooper, Seiford and Tone (2000) for introductory texts on nonparametric production and efficiency analysis.

[3] See, e.g., the 'subset rationalization' concept of Banker and Maindiratta (1988) and the 'goodness-of-fit' concept of Varian (1990), which essentially reconcile the neoclassical nonparametric production analysis literature (see, e.g., Afriat, 1972;

As for our specific methodology, rather than resorting to some parametric specification of the production technology, we use the mere technological postulates of nested input requirement sets (or free output disposability) and convexity in output space. Both technology properties have often been used in a nonparametric setting. For example, Varian (1984) suggests the assumption of nested input sets, while Petersen (1990) and Bogetoft (1996) suggest the use of convexity in output space. As we will discuss, these minimal assumptions allow for analyzing cost efficient behavior from the raw price and quantity data by exploiting specific features of production processes characterized by economies of scope.

The second feature of our approach pertains to this particular (scope economies) interpretation of our empirical cost efficiency conditions. More specifically, we take it that the very nature of scope economies lies in joint input use and input externalities. The cost rationalizing effect of joint input use for multiple output firms is evident. For instance, as for our own empirical application, senior researchers can serve as an input in the production of both academic publications and doctorates that are delivered by the research production unit. Within the same setting, input externalities occur when the presence of a distinguished scholar has beneficial effects on the productivity of other members of the research unit, even if she or he is not directly involved in the production of the associated research output. More generally, input externalities refer to cost saving (or productivity enhancing) effects to be attributed to inputs (employed by the same production unit) that are not used in a direct manner for the production of the output under consideration.

While the illustrative application in the current chapter pertains to the specific case of academic research production, it is worth stressing that scope economies prevail in a wide variety of real-life situations, in the public sector (e.g. public railway companies that simultaneously provide freight and passenger transport) as well as in the private sector (e.g. banks that also provide insurance services). More generally, given our starting position that scope economies (originating from jointly used inputs and input externalities) form the very economic motiva-

Hanoch and Rothschild, 1972; and Varian, 1984) and the *Data Envelopment Analysis* (DEA) literature. [The term DEA, which was introduced by Charnes, Cooper and Rhodes (1978), is often used for summarizing the literature on nonparametric efficiency analysis.]

tion for multi-output production, we believe that our methodology becomes a useful analytical tool as soon as the production behavior is characterized by multiple outputs.

Finally, our method does not *a priori* impose economies of scope; it does not assume any structure regarding the nature of the effects resulting from joint input use and input externalities. In addition, it does not presume that the empirical analyst knows which (parts of the) observed input quantities represent joint use or are attributed to specific outputs. Indeed, such non-observability of the input distribution is often the case in real-life applications. That is, although we can observe aggregate inputs (for example, the numbers of senior and junior researchers), it may be quite difficult to determine which inputs are directly associated with what output (academic publications or doctoral dissertations). In the concluding section, we indicate how the presented model can be refined in the case that such additional information regarding the input quantity distribution is available.

Still, even though we impose minimal a priori structure regarding the nature of the scope economies or the input distribution, we can derive testable conditions for cost efficient behavior. Interestingly, these conditions are expressed in terms of *observable* 'aggregate' price and quantity information. In fact, we also extend our tools for nonparametric cost efficiency analysis to apply when only input-output quantity and no price information is available. That is, there is no need to disaggregate the observed firm demand and supply data to analyze the firm's cost efficiency, which make our conditions easy to implement in practice.

To demonstrate its practical usefulness, we apply our methodology by assessing the cost efficiency of research programs in Economics and Business Management Faculties of Dutch universities. Our data cover the period 1996-2000 and were delivered by the universities in the context of the quinquennial assessment of university research conducted under the auspices of the Association of Dutch Universities (VSNU). As argued above, the multi-output research production is likely to be characterized by economies of scope, which makes this data set well fit to illustrate our methodology.

The rest of this chapter is organized as follows. Section 5.2 presents a nonparametric characterization of cost efficient production behavior under economies of scope. As we will discuss, this provides nonparametric necessary and sufficient conditions for efficient behavior that are expressed in terms of *unobservable* price and quantity information. Section 5.3 subsequently presents the corresponding necessary and sufficient cost efficiency conditions that solely use *observable* information, and which are easy to implement in practice. Section 5.4 presents our empirical application to research programs in Dutch Economics and Business Management Faculties. Section 5.5 summarizes and provides some concluding remarks regarding potential extensions and refinements of the presented methodology. The Appendix contains the proofs of our results.

5.2 Cost efficient production behavior under economies of scope: a nonparametric characterization

We consider firms (broadly defined) that use a vector of m inputs $\mathbf{x} \in \mathbb{R}^m_+$ to produce an s-valued output quantity $\mathbf{y} \in \mathbb{R}^s_+$; in the following, we let $K = \{1, \ldots, s\}$ denote the output index set. Next, we assume a data set with T firm observations; we use $S = \{1, \ldots, T\}$ to denote the corresponding index set. For each observation $i \in S$, we observe the output vector $\mathbf{y}^i$, the corresponding input vector $\mathbf{x}^i$ and the input price vector $\mathbf{p}^i \in \mathbb{R}^m_{++}$. [In a following step, we relax the assumption that input prices are observed.]

In what follows, we consider the most general production processes, which correspond to our interpretation of scope economies discussed in the introduction. Specifically, we take account of the fact that the production process of each output may be characterized by production externalities and joint input use. To do so, we consider *decomposed input vectors* $\widehat{\mathbf{x}} = (\mathbf{x}'_1 \cdots \mathbf{x}'_s\ \mathbf{x}'_{s+1})'$ for an aggregate input vector $\mathbf{x}$ such that

$$\mathbf{x} = \mathbf{x}_1 + \mathbf{x}_2 + \cdots + \mathbf{x}_{s+1} \text{ and } \mathbf{0} \leq \mathbf{x}_l \leq \mathbf{x} \text{ for } l = 1, \ldots, s+1.$$

In this specification, each subvector $\mathbf{x}_k$, contains the input quantities that are directly allocated to the production of the output k, while the remaining subvector $\mathbf{x}_{s+1}$ captures the jointly used input. . Evidently, each $\widehat{\mathbf{x}} = (\mathbf{x}'_1 \cdots \mathbf{x}'_s\ \mathbf{x}'_{s+1})'$ defines a unique $\mathbf{x} = \sum_{l=1}^{s+1} \mathbf{x}_l$; we will repeatedly use this in our following discussion.

Example 5.1. *To illustrate the concept, we consider a situation where three inputs ($m = 3$) are used for the production of two outputs ($s = 2$). Suppose a firm with input vector* $\mathbf{x} = (6\ 5\ 2)'$. *Given that we have two outputs, we can define the corresponding decomposed input vector as* $\widehat{\mathbf{x}} = (\mathbf{x}_1'\ \mathbf{x}_2'\ \mathbf{x}_3')'$ *; the components* $\mathbf{x}_1$ *and* $\mathbf{x}_2$ *then capture the input used for the respective outputs 1 and 2, and* $\mathbf{x}_3$ *contains the jointly used input. A feasible specification is* $\mathbf{x}_1 = (2\ 1\ 1)'$, $\mathbf{x}_2 = (3\ 2\ 1)'$ *and* $\mathbf{x} = (1\ 2\ 0)'$. *[Note that this specification effectively satisfies* $\mathbf{x} = \sum_{l=1}^{3} \mathbf{x}_l$ *but, of course, many other specifications are equally feasible.] In words, this specification implies that respectively 2 and 3 units of the input 1 are used for the production of the outputs 1 and 2, while 1 unit of the input 1 is jointly used; a directly similar interpretation holds for the inputs 2 and 3.*

At this point, it is worth stressing that we usually cannot observe the allocation of the observed input vector $\mathbf{x}$ to its constituent components $\mathbf{x}_1, \ldots, \mathbf{x}_{s+1}$; the mere restriction is that, for each observation i, the vectors $\mathbf{x}_k^i$ ($k = 1, \ldots, s + 1$) should sum up to the observed input vector $\mathbf{x}^i$. Note further that it may well be that some components of the decomposed vector $\widehat{\mathbf{x}}$ equal zero; e.g., there may be no joint input use, which means that $\mathbf{x}_{s+1}$ is a zero vector. [Such cases, which include additional information, may entail more stringent cost efficiency conditions. We return to this in the concluding discussion.]

We describe the technology in terms of input requirement sets $V_k(y_k) \subseteq (\mathbb{R}_+^m)^{s+1}$ associated with the k-th output quantity $y_k \in \mathbb{R}_+$; the statement $\widehat{\mathbf{x}} \in V_k(y_k)$ then indicates that the (decomposed) input vector $\widehat{\mathbf{x}}$ produces at least the output quantity y_k. This specification of the sets $V_k(y_k)$ effectively implies that the production of each output k may depend not only on the input $\mathbf{x}_k$ specific to the production of the k-th output but also, through production externalities, on the inputs $\mathbf{x}_{k^*}$ that are allocated to some other output k^*, and on the jointly used input $\mathbf{x}_{s+1}$. By construction, the input requirement sets are *nested* in the following sense:[4]

$$(\widehat{\mathbf{x}} \in V_k(y_k) \wedge y_k \geq \widetilde{y}_k) \Rightarrow \widehat{\mathbf{x}} \in V_k(\widetilde{y}_k);$$

this is a standard condition which reflects that less output never requires more input or, in other words, that outputs are freely disposable.

[4] As discussed by Varian (1984), including this condition avoids trivial rationalizations of the data in the sense of the following Definition 5.2.

The presence of production externalities and jointly used inputs makes it impossible to consider each output separately in the cost efficiency analysis; e.g., the specification of the (unobserved) $\widehat{\mathbf{x}} \in V_k(y_k)$ most clearly reveals that the input requirement sets associated with different outputs are mutually interdependent. To obtain a setting that allows for analyzing cost efficiency at the (multi-output) firm level, we define the set $\mathbf{V}(\mathbf{y})$, which contains all input vectors $\widehat{\mathbf{x}}$ that can produce the multi-valued output vector $\mathbf{y}$. The formal interrelationship between the input requirement set $\mathbf{V}(\mathbf{y})$ and the sets $V_k(y_k)$, is as follows:

$$\text{for } \mathbf{y} = (\, y_1 \cdots\ y_s \,)' :\ \widehat{\mathbf{x}} \in \mathbf{V}(\mathbf{y}) \Leftrightarrow \forall k \in K : \widehat{\mathbf{x}} \in V_k(y_k).$$

Using $\mathbf{V}(\mathbf{y})$, we can next define the further production assumption of *convexity in output space*. We define this property in terms of some given budget z that can be used for purchasing the inputs (under the prices $\mathbf{p}$):

$$\begin{aligned}&(\exists \widehat{\mathbf{x}}^a : \mathbf{p}'\mathbf{x}^a \leq z \wedge \widehat{\mathbf{x}}^a \in \mathbf{V}(\mathbf{y}^a)) \wedge (\exists \widehat{\mathbf{x}}^b : \mathbf{p}'\mathbf{x}^b \leq z \wedge \widehat{\mathbf{x}}^b \in \mathbf{V}(\mathbf{y}^b))\\ &\Rightarrow \forall\ \lambda \in [0,1] : (\exists \widehat{\mathbf{x}}^c : \mathbf{p}'\mathbf{x}^c \leq z \wedge \widehat{\mathbf{x}}^c \in \mathbf{V}(\lambda \mathbf{y}^a + (1-\lambda)\mathbf{y}^b)).\end{aligned}$$

In words, this definition states that, if the budget z can afford the production of $\mathbf{y}^a$ and $\mathbf{y}^b$ (through the (decomposed) input vectors $\mathbf{x}^a$ and $\mathbf{x}^b$, respectively) then it can also produce any convex combination of these output vectors (through some input vector $\mathbf{x}^c$). The economic interpretation of the condition is that the marginal rates of output transformation are everywhere decreasing (or, *stricto sensu*, non-increasing) along the boundary of the output producible set associated with the budget z.[5]

Using this convexity property, we can characterize efficient production behavior. As a preliminary step, we note that each input requirement set $V_k(y_k)$ corresponds to a production function $f_k(\widehat{\mathbf{x}})$, which gives the maximum quantity of output k that can be produced with $\widehat{\mathbf{x}}$. We have:

$$\widehat{\mathbf{x}} \in V_k(y_k) \Leftrightarrow f_k(\widehat{\mathbf{x}}) \geq y_k.$$

Similarly, we may relate the set $\mathbf{V}(\mathbf{y})$ to the production function $\mathbf{f}(\widehat{\mathbf{x}}) = (\, f_1(\widehat{\mathbf{x}}) \cdots\ f_s(\widehat{\mathbf{x}}) \,)'$.

[5] It can be verified that this output convexity condition is actually somewhat stronger than that forwarded by Petersen (1990) and Bogetoft (1996) in a similar context. The more stringent property is essential for obtaining the cost efficiency condition in (5.1), which will form the basis for our nonparametric characterization of multi-output production under economies of scope.

In our approach, cost efficient production behavior means that, for each firm observation i, the selected (decomposed) input vector $\widehat{\mathbf{x}}^i$ yields an output combination $\mathbf{y}^i$ that is situated on the efficient boundary of the (convex) set of producible output combinations associated with the given budget (which corresponds to the observed outlay $(\mathbf{p}^i)'\mathbf{x}^i$); this effectively represents a 'rational' allocation of the available budget. To formally define the condition, we (again) use the standard result in welfare economics that under convex utility possibility sets, any Pareto-efficient allocation can be characterized as a stationary point of a linear social welfare function (see, e.g., Mas-Colell, Whinston and Green, 1995). This result is readily translated towards the current setting, which is characterized by convex output producible sets (instead of utility possibility sets). More specifically, we obtain that efficient production behavior requires that each observation i maximizes a multi-output production function (linear in terms of $\mathbf{y}^i$) for the observed outlay, i.e. $\forall i \in S, \exists \mu^i \in \mathbb{R}^s_+$:

$$(\mu^i)'\mathbf{y}^i = \max_{\widehat{\mathbf{x}}}\{(\mu^i)'\mathbf{f}(\widehat{\mathbf{x}}) \mid \mathbf{x} \in \mathbb{R}^m_+ \text{ with } (\mathbf{p}^i)'\mathbf{x} \leq (\mathbf{p}^i)'\mathbf{x}^i\}. \tag{5.1}$$

In this cost efficiency condition, the entries of $\mu^i \in \mathbb{R}^s_+$ can be interpreted as 'priority weights', which the firm under evaluation attributes to the different outputs. These weights correspond to some implicit (possibly nonlinear) production objective function that aggregates the different individual outputs, and which underlies the observed output choices. Importantly, these priority weights need not be constant across the firm observations: they can change depending on the economic circumstances. For example, if the valuation of some output k increases, then the firm may want to increase (in relative terms) the production of output k, which is translated in a higher priority weight. The analogy with the Pareto efficiency concept in welfare economics is immediate: cost-efficient behavior, for a given outlay, implies that it is impossible to increase output k without decreasing output k^* $(k^* \neq k)$.

The question of data consistency with the cost efficiency condition is then whether it is possible to conceive a collection of input requirement sets that makes observed behavior consistent with the above efficiency condition. The following definition states the formal conditions for such a *cost rationalization* (or *C-R*) of the production data:

Definition 5.2. *For a production process characterized by production externalities and joint input use, a family of input requirement sets* $\{V_k(y_k),\ k \in K\}$ *provides a* C-R *of the set* $\{(\mathbf{p}^i, \mathbf{x}^i, \mathbf{y}^i) \mid i \in S\}$ *if there exists a production function* $\mathbf{f}(\widehat{\mathbf{x}})$*, such that for each* $i \in S$ *there exists a decomposed input vector* $\widehat{\mathbf{x}}^i = (\,(\mathbf{x}_1^i)' \cdots\ (\mathbf{x}_s^i)'\ (\mathbf{x}_{s+1}^i)'\,)'$ *and a priority weight vector* $\mu^i \in \mathbb{R}_+^s$ *that satisfy:*

(i) $\mathbf{f}(\widehat{\mathbf{x}}^i) = \mathbf{y}^i$*;*

(ii) $(\mu^i)'\mathbf{f}(\widehat{\mathbf{x}}^i) \geq (\mu^i)'\mathbf{f}(\widehat{\mathbf{x}})$ *for all* $\widehat{\mathbf{x}} = (\,(\mathbf{x}_1)' \cdots (\mathbf{x}_s)'\ (\mathbf{x}_{s+1})'\,)' \in (\mathbb{R}_+^m)^{s+1}$ *with* $\mathbf{p}_i'(\sum\limits_{k=1}^{s+1} \mathbf{x}^k) \leq \mathbf{p}_i'\mathbf{x}_i$.

In words, this definition requires that there must exist at least one feasible decomposition of the observed input vectors $\mathbf{x}^i$ into $\widehat{\mathbf{x}}^i$ and, accordingly, priority weight vectors μ^i and some production function $\mathbf{f}(\widehat{\mathbf{x}})$ such that each firm observation i is consistent with the efficiency condition (5.1). This definition is clearly analogous to the Definition 2.1.

This efficiency condition cannot be used as such, since the production technology (and, hence, the function $\mathbf{f}(\widehat{\mathbf{x}})$) is typically unknown. Essentially, the nonparametric approach to analyzing production behavior forwards efficiency conditions that do not necessitate a (non-verifiable) functional specification of the production possibilities. To provide a nonparametric characterization of cost efficient behavior in the sense of Definition 5.2, we first define the additional concept of implicit price vectors $(\widehat{\mathbf{p}}_1, \ldots, \widehat{\mathbf{p}}_s)$ for an (aggregate) input price vector $\mathbf{p}$ as

$$\begin{aligned}&\forall k \in K : \widehat{\mathbf{p}}_k = (\mathfrak{p}_{k,1}' \cdots \mathfrak{p}_{k,s}'\ \mathfrak{p}_{k,s+1}')' \text{ such that}\\&\forall l = 1, \ldots, s+1 : \mathfrak{p}_{k,l} \in \mathbb{R}_+^m \text{ and } \mathfrak{p}_{1,l} + \cdots + \mathfrak{p}_{s,l} = \mathbf{p}.\end{aligned}$$

This concept complements the earlier concept of decomposed input vector $\widehat{\mathbf{x}}$: each $\widehat{\mathbf{p}}_k = (\mathfrak{p}_{k,1}' \cdots \mathfrak{p}_{k,s}'\ \mathfrak{p}_{k,s+1}')'$ captures the fraction of the price for the decomposed input quantities $\widehat{\mathbf{x}}^i = (\,(\mathbf{x}_1^i)' \cdots\ (\mathbf{x}_s^i)'\ (\mathbf{x}_{s+1}^i)'\,)'$ that is attributed to the output k. To see this, we first recall that the decomposition of the (aggregate) input vector x into $s+1$ components effectively reveals the different channels through which the observed inputs are allocated. Correspondingly, each l-th component $\mathfrak{p}_{k,l}$ of the implicit price vectors $\widehat{\mathbf{p}}_k$ gives the fraction of the price of each input component $\mathbf{x}_l$ that is attributed to the output k.[6] More specifically, the components $\mathfrak{p}_{k,k}$ capture the fraction of the price of the input directly allocated to output k that is effectively borne by that output k. Next, the components $\mathfrak{p}_{k,k^*}$ ($k^* \in K, k^* \neq k$) refer to the possibility of input externalities: $\mathfrak{p}_{k,k^*} \neq \mathbf{0}$ means that the inputs allocated to the output k^* (i.e. $\mathbf{x}_k^*$) benefit the production of the output k (which is thus compensated through $\mathfrak{p}_{k,k^*}$). Finally, as for the jointly used input $\mathbf{x}_{s+1}$, the cost must be distributed over the different outputs (see in particular $\mathfrak{p}_{1,s+1} + \cdots + \mathfrak{p}_{s,s+1} = \mathbf{p}$).

Example 5.3. *To further illustrate the concept, we recapture the situation with three inputs and two outputs in Example 5.1. Suppose an input price vector* $\mathbf{p} = (2\ 1\ 3))'$. *Given that we have two outputs, we can define implicit price vectors* $\widehat{\mathbf{p}}_1 = (\mathfrak{p}_{1,1}'\ \mathfrak{p}_{1,2}'\ \mathfrak{p}_{1,3}')'$ *and* $\widehat{\mathbf{p}}_2 = (\mathfrak{p}_{2,1}'\ \mathfrak{p}_{2,2}'\ \mathfrak{p}_{2,3}')'$. *A feasible specification is the following:*

$$\mathfrak{p}_{1,1} = \begin{pmatrix} 1.5 \\ 0.5 \\ 1.5 \end{pmatrix}, \mathfrak{p}_{1,2} = \begin{pmatrix} 0 \\ 0.5 \\ 2 \end{pmatrix}, \mathfrak{p}_{1,3} = \begin{pmatrix} 1 \\ 0 \\ 1.5 \end{pmatrix} \text{ and}$$
$$\mathfrak{p}_{2,1} = \begin{pmatrix} 0.5 \\ 0.5 \\ 1.5 \end{pmatrix}, \mathfrak{p}_{2,2} = \begin{pmatrix} 2 \\ 0.5 \\ 1 \end{pmatrix}, \mathfrak{p}_{2,3} = \begin{pmatrix} 1 \\ 1 \\ 1.5 \end{pmatrix}.$$

In words, $\mathfrak{p}_{1,1} \neq \mathbf{p}$ *implies that the output* 1 *does not fully bear the cost for the input used for its production* ($\mathbf{x}_1$)*; this reflects that input externalities benefit the production of the output* 2 *(for which the compensation is captured by* $\mathfrak{p}_{2,1}(= \mathbf{p} - \mathfrak{p}_{1,1})$*). Similarly, the input used for the production of the output* 2 *(*$\mathbf{x}_2$*) implies production externalities towards the production of the output* 1 *(see the specification of* $\mathfrak{p}_{1,2}$ *and* $\mathfrak{p}_{2,2}$*); in this case there are no externalities associated with the input* 1 *(as the first entry of* $\mathfrak{p}_{1,2}$ *is zero). Finally, the remaining components* $\mathfrak{p}_{1,3}$ *and* $\mathfrak{p}_{2,3}$ *distribute the cost of the jointly used input.*
As a final note, we indicate that a specification of the decomposed input vector $\widehat{\mathbf{x}}$ *and the implicit price vectors* $(\widehat{\mathbf{p}}_1, \ldots, \widehat{\mathbf{p}}_s)$ *effectively allows for computing the cost share attributed to each output k (as* $\widehat{\mathbf{p}}_k'\widehat{\mathbf{x}}$ *). As for our example, the given specification of* $\widehat{\mathbf{x}}$ *(in Example 5.1),* $\widehat{\mathbf{p}}_1$ *and* $\widehat{\mathbf{p}}_2$ *implies a cost level of 9 (*$=\widehat{\mathbf{p}}_1'\widehat{\mathbf{x}} = \mathfrak{p}_{1,1}'\mathbf{x}_1 + \mathfrak{p}_{1,2}'\mathbf{x}_2 + \mathfrak{p}_{1,3}'\mathbf{x}_3$*) for the output* 1 *and a cost level of 14 (*$=\widehat{\mathbf{p}}_2'\widehat{\mathbf{x}} = \mathfrak{p}_{2,1}'\mathbf{x}_1 + \mathfrak{p}_{2,2}'\mathbf{x}_2 + \mathfrak{p}_{2,3}'\mathbf{x}_3$ *) for the*

[6] In fact, as in Chapter 2, the intuition of the implicit price vectors is analogous to that of Lindahl prices in the context of public goods

output 2. Note that the sum of these cost shares equals the total cost of production $\mathbf{p}'\mathbf{x}$.

Once more we should stress that, for a given observation i, we usually cannot observe the exact specification of the implicit price vector $\widehat{\mathbf{p}}_k^i = ((\mathfrak{p}_{k,1}^i)' \ \cdots \ \mathfrak{p}_{k,s}^i)' \ \mathfrak{p}_{k,s+1}^i)')'$ associated with each output k. The only restriction is that the components $\mathfrak{p}_{k,l}^i$ must add up (over the different outputs k) to the observed (aggregate) price, i.e. $\mathfrak{p}_{1,l}^i + \cdots + \mathfrak{p}_{s,l}^i$. We return to the use of additional information regarding the specification of the implicit price vectors in the concluding discussion.

We are now in a position to state a nonparametric necessary and sufficient condition for cost efficient production behavior in the sense of Definition 5.2[7]

Proposition 5.4. *For a production process characterized by production externalities and joint input use, there is a family of closed, convex, positive monotonous input requirement sets that provide a* C-R *of the data if and only if for each* $i \in S$ *there exists*
(i) a decomposed input vector $\widehat{\mathbf{x}}^i = ((\mathbf{x}_1^i)' \cdots (\mathbf{x}_s^i)' \ (\mathbf{x}_{s+1}^i)')'$ *and*

(ii) implicit price vectors $(\widehat{\mathbf{p}}_1^i, \ldots, \widehat{\mathbf{p}}_s^i)$,

such that $\forall i, j \in S, k \in K : y_k^j \geq y_k^i \Rightarrow (\widehat{\mathbf{p}}_k^i)'\widehat{\mathbf{x}}^j \geq (\widehat{\mathbf{p}}_k^i)'\widehat{\mathbf{x}}^i$.

Hence, consistency with the cost efficiency condition requires that observed behavior satisfies a number of cost minimization conditions (i.e., one for each individual output k), which are expressed in terms of decomposed input vectors $\widehat{\mathbf{x}}^i$ and corresponding implicit prices $(\widehat{\mathbf{p}}_1^i, \ldots, \widehat{\mathbf{p}}_s^i)$ for each observation i. If observation j produces more of the output k than observation i ($y_k^j \geq y_k^i$), then cost efficiency requires that the k-th output cost (under the prices $\widehat{\mathbf{p}}_k^i$) for observation i does not exceed that for observation j (i.e.,$(\widehat{\mathbf{p}}_k^i)'\widehat{\mathbf{x}}^j \geq (\widehat{\mathbf{p}}_k^i)'\widehat{\mathbf{x}}^i$). Interestingly, these cost minimization conditions are formally analogous to the condition derived by Varian (1984; Theorem 1) in the single output case. Given our multi-output orientation, we identify a separate condition for each entry $k \in K$ of the evaluated output vector $\mathbf{y}^i$. Our above explanation of the vectors $\widehat{\mathbf{x}}^i$ and $(\widehat{\mathbf{p}}_1^i, \ldots, \widehat{\mathbf{p}}_s^i)$ makes clear that this

[7] In Proposition 5.4, *positive monotonicity* of a set $V_k(y_k)$ means that $(\widehat{\mathbf{x}} \in V_k(y_k) \wedge \widehat{\mathbf{x}}^* \geq \widehat{\mathbf{x}}) \Rightarrow \widehat{\mathbf{x}}^* \in V_k(y_k)$. This is essentially the property of free input disposal, i.e. more input can always produce the same output.

nonparametric characterization has a natural intuition in terms of the underlying model of joint input use and production externalities.

5.3 Empirical tests of cost efficient behavior under economies of scope

The C-R condition in Proposition 5.4 is expressed in terms of decomposed quantity vectors $\widehat{\mathbf{x}}^i$ and implicit price vectors $(\widehat{\mathbf{p}}_1^i, \ldots, \widehat{\mathbf{p}}_s^i)$. Empirical testing of the condition would be easy if these vectors were observed. However, as in the previous chapters, such price and quantity information is usually not available, which makes direct empirical implementation of the C-R conditions generally infeasible in computational terms. Specifically, one cannot conclude whether a C-R is possible for a given data set by using an algorithm that tries to find unobserved implicit prices and decomposed input vectors that are consistent with the conditions in Proposition 5.4: in general, no algorithm can exhaust all possible values and combinations of these unobservables. Therefore, we next institute necessity and sufficiency conditions that solely include the 'aggregate' observed price and quantity information.

We first present the necessity condition. Before formulating that condition, we define an *output dominating reference set* R^i for observation i as

$$R^i = \{j^k \in S | \forall k \in K, \exists j^k : y_k^{j^k} \geq y_k^i\}.$$

In words, each set R^i is constructed such that, for each output k, it contains at least one observation $j^k \in R^i$ that dominates observation i in that output k ($y_k^{j^k} \geq y_k^i$). For each observation i,

$\mathbf{R}^i$ is the collection of output dominating reference sets R^i.

Example 5.5. *We illustrate by means of three observations that are taken from our own empirical application (see Section 5.4). It considers a situation with three outputs ($s = 3$) and we have:*

$$\mathbf{y}^1 = \begin{pmatrix} 1 \\ 27 \\ 4 \end{pmatrix}, \mathbf{y}^2 = \begin{pmatrix} 1 \\ 27 \\ 2 \end{pmatrix} \textit{ and } \mathbf{y}^3 = \begin{pmatrix} 0 \\ 13 \\ 5 \end{pmatrix}.$$

Let us construct the output dominating reference sets for observation 1*. Trivially, one such set is the singleton* $\{1\}$*. Evidently, this implies* *that the sets* $\{1,2\},\{1,3\}$ *and* $\{1,2,3\}$ *equally satisfy the definition of output dominating reference set for observation* 1*. [More generally, we have that, if* $R_1^i \subseteq S$ *is an output dominating reference set for observation* i*, then by construction any larger subset* R_2^i *(with* $R_1^i \subseteq R_2^i$*) is also an output dominating reference set for the same observation.] In addition, given the output vectors of observations* 2 *and* 3*, a final output dominating reference set is the pair* $\{2,3\}$ *(because* $y_1^2 \geq y_1^1, y_2^2 \geq y_2^1$ *and* $y_3^3 \geq y_3^1$*). Hence, we obtain the collection of output dominating reference sets*

$$\mathbf{R}^1 = \{\{1\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\}\}.$$

We can now define the nonparametric necessary condition for cost efficient production that solely includes aggregate price and quantity information:

Proposition 5.6. *For a production process characterized by production externalities and joint input use, there is a family of closed, convex, positive monotonous input requirement sets that provide a* C-R *of the data only if for each* $i \in S : (\mathbf{p}^i)'\mathbf{x}^i \leq \min\limits_{R^i \in \mathbf{R}^i} (\mathbf{p}^i)'(\sum\limits_{j \in R^i} \mathbf{x}^j)$.

This condition compares the cost level $(\mathbf{p}^i)'\mathbf{x}^i$ for the evaluated firm observation i to the cost level for each combination of observations $j^k \in S$ with $y_k^{j^k} \geq y_k^i$ for each output $k \in K$. More specifically, the condition states that the cost level of i should not exceed, under the prices that apply to i, the cost level for the summed input vector $\sum\limits_{j \in R^i} \mathbf{x}^j$ associated with any set $R^i \in \mathbf{R}^i$. Intuitively, if this condition were not met, then observation i could have produced (at least) the same output vector $\mathbf{y}^i$ at a lower cost level by using the (sum) input $\sum\limits_{j \in R^i} \mathbf{x}^j$ instead of the chosen vector $\mathbf{x}^i$.

If an observation i does not meet the corresponding necessary condition for cost efficient behavior, then it is useful to quantify the corresponding deviation from the (necessary) efficiency condition. To do so, we use a *cost efficiency measure* defined as the ratio of the minimum cost level needed to obtain consistency with the necessity requirement over the actual cost level:

$$\varphi^i = \frac{\min\limits_{R^i \in \mathbf{R}^i} (\mathbf{p}^i)'(\sum\limits_{j \in R^i} \mathbf{x}^j)}{(\mathbf{p}^i)'\mathbf{x}^i}.$$

The corresponding necessary condition for a *C-R* of the data to be possible is that $\varphi^i = 1$ for each $i \in S$. The value captures the extent to which costs should (minimally) be reduced in order to obtain consistency of the observation i with the cost rationalization conditions. From that perspective, the measure φ^i reveals for each individual firm observation the degree of consistency with the necessary *C-R* requirement in Proposition 5.6, and may thus be interpreted as a *goodness-of-fit* measure for the cost efficiency condition under investigation. See Varian (1990) and Färe and Grosskopf (1995) for a detailed discussion of this goodness-of-fit idea in a similar context of nonparametric production analysis.

Example 5.7. *To further illustrate, we recapture the situation in Example 5.5. Specifically, we evaluate observation* 1 *and use the additional information (again taken from our empirical application; see Section 5.4) that the three observations produce the three outputs by means of two inputs (*$m = 2$*) with quantities and prices*

$$\mathbf{x}^1 = \begin{pmatrix} 28.2 \\ 23 \end{pmatrix}, \mathbf{x}^2 = \begin{pmatrix} 7.2 \\ 9.2 \end{pmatrix} \mathbf{x}^3 = \begin{pmatrix} 10.2 \\ 8 \end{pmatrix} \text{ and } \mathbf{p}^1 = \begin{pmatrix} 1798.660 \\ 3129.605 \end{pmatrix}.$$

On the one hand, we find that $(\mathbf{p}^1)'\mathbf{x}^1 = 122933.115$*. On the other hand, it can be verified that*

$$\min_{R^1 \in \mathbf{R}^1} (\mathbf{p}^1)'(\sum_{j \in R^1} \mathbf{x}^j) = (\mathbf{p}^1)'(\mathbf{x}^2 + \mathbf{x}^3) = 85289.$$

Hence, $(\mathbf{p}^1)'\mathbf{x}^1 > \min_{R^1 \in \mathbf{R}^1} \mathbf{p}^1)'(\sum_{j \in R^1} \mathbf{x}^j)$ *and, thus, observation* 1 *does not meet the necessary cost minimization condition in Proposition 5.6. The corresponding cost efficiency measure* $\varphi^1 = \frac{85289}{122933.115} = 0.694$ *suggests that, for the given output, observation* 1 *can reduce its cost level by (at least)* 30.6%*. Obviously,* $\varphi^1 < 1$ *a fortiori implies that a* C-R *of this data set is impossible.*

Proposition 5.6 institutes a condition on the observable price and quantity information that should always be met by production processes consistent with the *C-R* Definition 5.2. Still, meeting this condition does not mean that there effectively exists a *C-R* of the production data under consideration; i.e. the condition is necessary but not sufficient for a *C-R* to be possible. A complementary sufficiency condition is:

Proposition 5.8. *For a production process characterized by production externalities and joint input use, there is a family of closed, convex, positive monotonous input requirement sets that provide a* C-R *of the data if it is possible to construct a partitioning* S_k $(k \in K)$ *with* $\bigcup_{k \in K} S_k = S$ *and* $S_k \cap S_{k^*} = \emptyset$ $(k, k^* \in K, k \neq k^*)$ *such that for* $i \in S_k$ *:*
(i) $\forall j \in S_k : y_k^j \geq y_k^i \Rightarrow (\mathbf{p}^i)'\mathbf{x}^i \leq (\mathbf{p}^i)'\mathbf{x}^j$ *and*
(ii) $\forall j^* \in S_{k^*}(k^* \neq k) : y_{k^*}^{j^*} > y_{k^*}^i$.

Intuitively, this condition considers the extreme scenario where each firm observation $i \in S_k$ allocates all inputs exclusively to the production of a single output k. The first part of the closing cost minimization condition then states that it should not be possible to produce (at least) the associated output quantity y_k^i at a lower cost when compared to any (similarly specialized) production plan j with $y_k^j \geq y_k^i$. The second part of the closing condition imposes that, under such exclusive allocation (or specialization), the observation i should not dominate some other observation j^* in output k^* if the latter observation effectively specializes in producing k^*.[8]

Clearly, testing data consistency with the empirical requirements in Propositions 5.6 and 5.8 is a finite process because, essentially, the cardinality of the set of observations j that dominate observation i in at least one output k is finite in nature (i.e. $\{j \in S \mid \exists k \in K : y_k^j \geq y_k^i\}$ is finite in nature). Given this, the necessity and sufficiency tests may be implemented by means of simple enumeration algorithms, which consecutively consider all possible specifications of the sets R^i for each observation i (for the necessity requirement) and S_k for each output k (for the sufficiency requirement). The next section provides an illustration for $s = 3$.

Note that these algorithms are, in comparison with the algorithms for testing the collective consumption model in Chapter 3, are much less computationally cumbersome. Indeed, in the production setting we observe the ordering information (i.e. the outputs y_k^i), while in the con-

[8] To avoid a conflict with the usual 'no free lunch' assumption, one may also interpret the sufficiency condition in terms of *quasi*-exclusive input allocation (i.e. the production of a single output consumes *almost* all inputs while a minimal amount of input is allocated to each other output). [The proof in the Appendix is easily accommodated.] More generally, it is worth stressing that, for data that are consistent with the sufficiency condition, this may not be the only data rationalizing interpretation. The sole implication of the sufficiency result is that (quasi-)exclusive input allocation *always* constitutes a possible interpretation.

sumption setting we have to reconstruct this ordering information (i.e. the revealed preferred relations R^m). As discussed in Chapter 3, it is this reconstruction that could be time consuming.

Again as in Chapter 2 our empirical necessity and sufficiency requirements will in general not coincide; this discrepancy essentially reflects the unobservability of the implicit prices and quantities in Proposition 5.4. The only instance in which both conditions are equivalent occurs when there is a single output (i.e. $s = 1$). The intuition is straightforward: in that case, the implicit input prices and quantities are the observed prices and quantities, and thus the necessity and sufficiency conditions for a *C-R* of the data always coincide; or, from a different perspective, the empirical implications of joint input use and production externalities become irrelevant if there is only a single output.

In the general case (for $s \geq 2$), violation of the necessary condition in Proposition 5.6 means that a *C-R* of the data is impossible, while consistency with the sufficient condition in Proposition 5.8 entails the opposite conclusion. As for data that meet the necessity but not the sufficiency condition, we cannot directly tell from the observable price and quantity information whether a *C-R* of the data is effectively possible. In such cases, one may, for example, impose some additional prior structure on the implicit input prices and quantities.

Still, even though the necessary condition should not generally coincide with the sufficient condition, we (again) may expect the two conditions to become equally powerful (or 'converge') when the sample size increases. Specifically, for large T the probability increases that for $i \in S$ there exists $k \in K$ such that $\forall k^* \in K, k^* \neq k$, we have $\min\limits_{j \in S}\{(\mathbf{p}^i)'\mathbf{x}^j \mid y^j_{k^*} \geq y^i_{k^*}\}$ gets closer to zero. In such a situation, the difference diminishes between

$$\min_{R^i \in \mathbf{R}^i} \sum_{j \in R^i} (\mathbf{p}^i)'\mathbf{x}^j \text{ and } \min_{j \in S: y^j_k \geq y^i_k} (\mathbf{p}^i)'\mathbf{x}^j.$$

As a result, the requirement $(\mathbf{p}^i)'\mathbf{x}^i \leq \min\limits_{R_i \in \mathbf{R}_i} \sum_{j \in R^i} (\mathbf{p}^i)'\mathbf{x}^j$ in Proposition 5.6 will approach $(\mathbf{p}^i)'\mathbf{x}^i \leq (\mathbf{p}^i)'\mathbf{x}^j$ for $j \in S$ such that $y^j_k \geq y^i_k$ in Proposition 5.8.[9]

[9] As for this last formulation of the sufficiency condition in Proposition 5.8, we note that the requirement $(\mathbf{p}^i)'\mathbf{x}^i \leq (\mathbf{p}^i)'\mathbf{x}^j$ for $j \in S_k : y^j_k \geq y^i_k$ is equivalent with $(\mathbf{p}^i)'\mathbf{x}^i \leq (\mathbf{p}^i)'\mathbf{x}^j$ for $j \in S$ such that $y^j_k \geq y^i_k$ under the stated condition that

The associated 'convergence rate' depends (positively) upon the input price-quantity variation in the data and, hence, we may expect it to increase with the number of inputs. For a given number of inputs, the speed of convergence will vary with the specific data generating process that underlies the aggregate production data, which in turn depends on the specific characteristics of the production process (see the function $\mathbf{f}(\widehat{\mathbf{x}})$ and the weighting vector μ^i in (5.1)). But, in general, we can safely argue that, for larger samples, the empirical implications of the fairly rudimentary allocation process underlying the sufficient condition will get closer to those of any more refined allocation process captured by the necessary condition.

So far, we have assumed that prices $\mathbf{p}^i$ for each firm observation i are known. In many cases, such reliable price information is not available.[10] Starting from Proposition 5.6, we may then formulate a necessary condition for a *C-R* of the data, as follows:

Corollary 5.9. *For a production process characterized by production externalities and joint input use, there is a family of closed, convex, positive monotonous input requirement sets that provide a* C-R *of the data only if there exists a collection of price vectors* $\{\mathbf{p}^i \in \mathbb{R}^m_+ \backslash \{\mathbf{0}\} \mid i \in S\}$ *such that*

$$\forall i \in S : (\mathbf{p}^i)'\mathbf{x}^i \leq \min_{R_i \in \mathbf{R}_i} (\mathbf{p}^i)'(\sum_{j \in R^i} \mathbf{x}^j).$$

The interpretation of the condition is as follows: in the absence of fully reliable price information, a necessary condition for data consistency with the *C-R* conditions is that there exists, for each observation i, at least one (non-zero) input price vector that implies consistency with the condition in Proposition 5.6. From the perspective of the evaluated production plan, such a price vector may be conceived as 'most favorable' in that it effectively minimizes the cost inefficiency. In a certain sense, such an implicit (most favorable) price vector may be interpreted as a *shadow price vector* that supports cost efficient behavior of the evaluated production vector.

$\min_{j \in S}\{(\mathbf{p}^i)'\mathbf{x}^j | y^{j^*}_{k^*} > y^i_{k^*}\} = 0$ for all $k^* \in K, k^* \neq k$. This follows from $\forall i \in S_k$: $(j^* \in S_{k^*}, k^* \neq k \Rightarrow y^{j^*}_{k^*} > y^i_{k^*})$; see also our proof of Proposition 5.8.

[10] See, e.g., Kuosmanen, Cherchye and Sipiläinen (2005) for a discussion of instances where reliable price information is not readily available. Our application in Section 5.4 contains a further example.

Checking consistency with the necessary condition in Corollary 5.9 boils down to solving the following linear programming problem for each observation i:

$$
\begin{array}{ll}
\textbf{Primal} & \textbf{Dual} \\
\theta^i = \max\limits_{u \in \mathbb{R}, \mathbf{p}^i \in \mathbb{R}^m_+} u & \theta^i = \min\limits_{\phi \in \mathbb{R}, \lambda^{R^i} \in \mathbb{R}_+} \phi \\
s.t. & s.t. \\
(\mathbf{p}^i)'\mathbf{x}^i = 1 & \phi \mathbf{x}^i \geq \sum\limits_{R^i \in \mathbf{R}^i} \lambda^{R^i} (\sum\limits_{j \in R^i} \mathbf{x}^j) \\
u \leq (\mathbf{p}^i)'(\sum\limits_{j \in R^i} \mathbf{x}^j) \ \forall R^i \in \mathbf{R}^i & \sum\limits_{R^i \in \mathbf{R}^i} \lambda^{R^i} = 1
\end{array}
\tag{5.2}
$$

In the primal formulation of the problem, the price normalization $(\mathbf{p}^i)'\mathbf{x}^i = 1$ effectively implements the condition $\mathbf{p}^i \in \mathbb{R}^m_+ \backslash \{\mathbf{0}\}$ in Corollary 5.9. Recalling our above interpretation of the corollary, the model checks whether, subject to the price normalization, there exists a set of implicit shadow prices that make the firm observation i consistent with the empirical cost minimization condition. Just like the measure φ^i that we defined before, the measure θ^i captures the degree of (shadow) cost efficiency of the observation i, and it can be interpreted as goodness-of-fit measure. Clearly, we have $\theta_i \geq \varphi^i$ and a necessary condition for cost efficient production behavior is that $\theta^i = 1$ for each observation i.

The dual problem in (5.2) computes (radial) Farrell (1957) efficiency with respect to a monotone production technology with convexified input sets; convexification is taken over the sets R^i. In fact, this primal-dual formulation shows the close connection with the nonparametric efficiency measurement literature known as *Data Envelopment Analysis* (DEA; after Charnes, Cooper and Rhodes, 1978). Specifically, the dual problem in (5.2) is formally similar to Bogetoft's (1996) DEA model that computes Farrell efficiency with respect to a similar production technology; the main difference is that we convexify over the summed input vectors $\sum\limits_{j \in R^i} \mathbf{x}^j$, which results from our specific scope economies perspective.[11] Our above discussion institutes the model as a tool for testing data consistency with (shadow) cost efficiency.

Example 5.10. *To illustrate, we recapture Example 5.7, but now we do not use the input price information. Again, we evaluate observa-*

[11] Hanoch and Rothschild (1972; Section 2) introduced a similar production technology representation as Bogetoft (1996) in a setting involving multiple inputs and a single output.

tion 1. For the given collection of output dominating reference sets $\mathbf{R}^i$ *(reported in Example 5.7), the problems in 5.2 take the form*

Primal	**Dual**
$\theta^1 = \max\limits_{u \in \mathbb{R}, \mathbf{p}^i \in \mathbb{R}^2_+} u$	$\theta^1 = \min\limits_{\phi \in \mathbb{R}, \lambda^i, i=1,\ldots,5} \phi$
$s.t.$	$s.t.$
$(\mathbf{p}^1)'\mathbf{x}^1 = 1$	$\phi\mathbf{x}^1 \geq \lambda^1(\mathbf{x}^1) + \lambda^2(\mathbf{x}^1 + \mathbf{x}^2)$
$u \leq (\mathbf{p}^1)'(\mathbf{x}^1)$	$+\lambda^3(\mathbf{x}^1 + \mathbf{x}^3) + \lambda^4(\mathbf{x}^2 + \mathbf{x}^3)$
$u \leq (\mathbf{p}^1)'(\mathbf{x}^1 + \mathbf{x}^2)$	$+\lambda^5(\mathbf{x}^1 + \mathbf{x}^2 + \mathbf{x}^3)$
$u \leq (\mathbf{p}^1)'(\mathbf{x}^1 + \mathbf{x}^3)$	$\lambda^1 + \lambda^2 + \lambda^3 + \lambda^4 + \lambda^5 = 1$
$u \leq (\mathbf{p}^1)'(\mathbf{x}^2 + \mathbf{x}^3)$	
$u \leq (\mathbf{p}^1)'(\mathbf{x}^1 + \mathbf{x}^2 + \mathbf{x}^3)$	

The outcome is $\theta^1 = 0.748$*. This means that, even when using the most favorable prices (in casu the computed shadow price vector* $(\mathbf{p}^i)' = (0.000\ 0.043)$ *), we can identify a potential cost reduction of (at least)* 25.2% *for the observation* 1.[12] *The corresponding minimum cost level is associated with the (sum) input vector* $(\mathbf{x}^2 + \mathbf{x}^3)$ *or, for the dual problem with* $\lambda^4 = 1$*.*

To conclude this illustration, we note that the specification of the set $\mathbf{R}^i$ *may be fine-tuned to enhance the efficiency of the empirical testing of the condition. For example, we may exclude from consideration an output dominating reference set* $R^i_2 \in \mathbf{R}^i$ *if there exist another set* $R^i_1 \in \mathbf{R}^i$ *such that* $R^i_1 \subseteq R^i_2$ *as, evidently,* $\mathbf{p}'(\sum\limits_{j \in R^i_2} \mathbf{x}^j) \geq \mathbf{p}'(\sum\limits_{j \in R^i_1} \mathbf{x}^j)$ *for any input price vector* $\mathbf{p}$*. For the specific data structure under investigation, this means that we can effectively restrict attention to* $\{\{1\}, \{2, 3\}\} \subset \mathbf{R}^1$ *when evaluating observation* 1*. As a matter of fact, we have used this insight for the computations of our own application presented in the next section.*

As a final note, we indicate that the computed shadow price vector $\mathbf{p}^i$ in 5.2 (and, correspondingly, the λ^{R^i} in the dual problem) should in

[12] The zero shadow price for the first input in this example is commonly referred to as a 'slack problem' in the DEA literature (e.g., Cooper, Seiford and Tone, 2000). In this respect, we indicate that the problems in (5.2) may be enriched by adding additional restrictions on the relative price that incorporate *a priori* information regarding feasible/realistic ranges for the endogenously defined prices (which can *inter alia* exclude zero shadow prices). In fact, such price restrictions have received considerable attention in the DEA literature. For compactness, we will not consider such restrictions in the following illustrative application.

general not be unique: there may be multiple input price vectors that support the same cost efficiency level. Therefore, we choose not to focus on these shadow price estimates in our following empirical application.

5.4 Illustrative application

We apply the presented methodology for examining the behavior of research programs in Economics and Business Management faculties of Dutch universities. Specifically, we evaluate the efficiency of 77 research programs organized at 8 universities. The same data set was studied by Cherchye and Vanden Abeele (2005), who motivate efficiency assessment within this setting by the argument that efficient research production is not guaranteed by the usual market correction mechanisms. These authors further claim that a cost efficiency evaluation model is particularly appropriate within this application context. But they focus on a different cost efficiency criterion, which does not explicitly incorporate the empirical implications of joint input use and production externalities. Still, as we argued in the introduction, the production process of university research seems well-suited to illustrate our method for assessing cost efficiency under scope economies. [In fact, this method implies a strengthened efficiency test as compared to that used by Cherchye and Vanden Abeele; we return to this below.]

Generally, a research program can be defined as "a group of researchers who join forces to investigate a particular theme, and in the process to educate researchers and to publish research results". Cherchye and Vanden Abeele argue that this definition institutes research programs as the natural production units for studying academic research efficiency. Building on that definition, they suggest the following input-output selection for characterizing the production of each program:

Inputs: (1) junior research staff (=PhD candidates), (2) senior research staff (= other research personnel). Following Cherchye and Vanden Abeele, we relate the output of each year to the sum of the inputs used in that same year and the inputs used in the two preceding years; this corrects for the fact that output in a particular year may actually result (at least partly) from inputs that have been used in preceding years.

Outputs: (1) total number of doctoral dissertations, (2) total number of refereed articles in top international journals, and (3) total number of refereed articles in international journals.

The input and output data are taken from the 'Quality Assessment Reports on Research 1996-2000', delivered by each Dutch university in the context of the quinquennial assessment by the VSNU (i.e., the Dutch association of universities). For each research program we have complete data for the years 1998, 1999 and 2000. Pooling the three cross-sections in the same sample, we have 229 observations in total.[13]

Cherchye and Vanden Abeele (2005) provide a detailed discussion about the data and the input-output selection. At this point, two special features of the input-output data deserve some additional explanation. First, the input data account for differences in the allocation of faculty time across different research programs. Specifically, they correct for differences in time spent on teaching in different universities and/or professional ranks (see in particular the discussion on p. 501 in Cherchye and Vanden Abeele). Second, outputs 2 and 3 count publications at the level of the research programs, which effectively avoids double counting publications that are co-authored by researchers of one and the same research program. The particular specification of these outputs (which includes top-journal publications in the output 2 as well as the output 3), entails an implicit extra premium for the top-journal publications. In other terms, it imposes the natural assumption that these publications get a higher weight than other refereed publications in international journals. More specifically, it implies that one input-output combination is a possible comparison partner for another input-output combination only if it produces at least the same amount of articles in international top journals (see the output 2) and, *in addition*, at least the same amount of articles in refereed journals in general (including top journals; see the output 3). This effectively imposes that a top publication can substitute for another (non-top) publication, but not *vice versa*.[14]

[13] Recall that the output in a given year is related to the sum of the inputs of that year and those of the two preceding years. Given this, Cherchye and Vanden Abeele use information on 79 research programs. Because of our specific focus on efficiency differences between research programs within specific specialization areas, we restrict attention to the 77 research programs for which the specialization type is known; this leaves 231 observations (= 77 programs x 3 years). From that sample, we further exclude two cases with important missing information, which eventually obtains 229 observations.

[14] Cherchye and Vanden Abeele (2005, p. 501-502) illustrate by means of a simple numerical example. In fact, while our data set implies a two-tiered classification of international journal publications, the same procedure can be used for introducing a three-tiered classification (e.g. 'top journals', 'very good journals' and

As a final note, we remark that the testing tools employed below require *stricto sensu* that the efficiency estimates are independently distributed. This assumption may be criticized as the input values for the years 1998, 1999 and 2000, which are used for computing the efficiency values, are interdependent by construction for each research program. In addition, and probably more importantly, efficiency values are obtained from comparison with a production possibility set that is constructed by means of a common set of reference units; i.e. the observed set of research programs. From that perspective, our (illustrative) test results below should be considered as 'indicative' rather than 'conclusive'. Still, as for our test results based on the full sample of 229 observations (see Table 5.1), we may refer to the consistency results that have been established for nonparametric efficiency analysis models similar to the one applied here, which suggest that this interdependency problem diminishes for sufficiently large samples (see, e.g., Banker, 1993, and Simar and Wilson, 1998). In this context, it is also worth referring to our discussion in the concluding section, on possible solutions for the sampling problem of DEA analyses.

5.4.1 Overall differences between universities and specialization types: observed prices

We focus on the necessity condition for cost efficient behavior under economies of scope (see Proposition 5.6). This necessity condition seems a natural starting point, since inconsistency with this condition implies *a fortiori* that the sufficiency condition cannot be met. In this respect, it is worth noting that the following results in Tables 5.1 and 5.2 imply that, for each of our exercises, the necessity condition in Proposition 5.6 is nowhere met at the sample level, which implies redundancy of testing the sufficiency condition in Proposition 5.8. Still, it is worth stressing that, while we will not illustrate this, testing the sufficiency condition is just as simple as testing the necessity condition; recall our discussion in Section 5.3 on the possibility of using enumeration algorithms.

For an observation that does not meet the necessary condition for cost efficient behavior, we quantify the degree of cost inefficiency by means

'other journals') or any other multi-tiered classification if that would seem recommendable. For the sake of compactness, we abstract from exploring this in our (illustrative) application. Still, we believe that our main qualitative results are fairly robust with respect to such additional journal classifications, which basically imply a more refined output structure.

of the measure φ that was introduced before. Apart from the observed research output and input quantities, this cost efficiency measure also needs input prices. For the different years that we consider, we take the price/wage information from the salary tables that were applicable to the Dutch universities at that time.[15] One problem in this respect is that salaries depend on the different types of staff (e.g., assistant professor, associate professor, etc.) and seniority. The VSNU-data do not allow us to determine the shares of the different types of research staff that are engaged in a certain university or a research program. Therefore, we assume that all junior researchers have the salary of a third year teaching assistant, while senior researchers are assumed to be of the associate professor level. Note that all price information included in the analysis is in real terms; we constructed real wages on the basis of the Eurostat harmonized consumer price index for the Netherlands. Consistent with our construction of the input quantities (i.e., the output of each year is related to the sum of the input in that same year and the input in the two preceding years) we evaluate the inputs by averaging real wages over the three input years associated with each output year. This yields the following relative input prices (i.e., the ratio of senior staff wage over junior staff wage): 1.745516 for the year 1998, 1.745468 for the year 1999, and 1.745524 for the year 2000. At this point, we note that our above 'simplifications of a complex reality' may lead critics to question the reliability of the prices that we use. Therefore, referring to our earlier discussion of Corollary 5.9 (and the corresponding linear programming problems in 5.2), we will also use shadow prices in a further exercise, which effectively corrects for potential 'unreliability' of the actual price information.

To begin, we focus on test results for (1) universities (i.e., the corresponding faculties of Economics and Business Management) as a whole and (2) specialization areas as a whole. Both exercises start from efficiency results based upon comparison of each individual research program (in the years 1998, 1999 and 2000) to the full set of 229 research program observations.[16] The first exercise then checks (significant) dif-

[15] Dutch universities are subject to a collective agreement that settles working conditions of university personnel. Since 2005, the same collective agreement applies to both public universities and the so-called special universities. Before 2005, there were differences in the agreement for both types of universities. However, the salary settlements were always the same.

[16] An alternative exercise could have considered a production setting with 30 outputs per university observations, i.e. 3 outputs for each of the 10 specialization

ferences in mean efficiency between research programs associated with different universities (while not correcting for different compositions in terms of specialization areas across universities). The second exercise similarly considers (significant) differences in mean efficiency between research programs that are active in different specialization domains (while not correcting for the identity of the organizing university).

Table 5.1 reports the efficiency results for the newly proposed methodology (in the column 'scope economies' cost efficiency). The table shows mean efficiency values for the different universities and specialization areas in the VSNU-data, respectively; for each (row) category of research programs, the table additionally reports (in the column 'p-value') the probability that the mean efficiency of research programs in that category equals the mean efficiency over all other categories. Focusing on the upper panel of the table, it is clear that there are rather important efficiency differences between the 8 universities. For example, there is a difference of more than 30 percentage points between the two extremes in the sample: Tilburg University obtains a mean efficiency value of about 67.2%, while the University of Nijmegen obtains a mean efficiency of only 35.6% in the period 1998-2000. [Remark, though, that the efficiency value of the latter university is based upon a rather small sample size.] Overall, the average efficiency level equals 52.1%. The top three universities in terms of mean efficiency values are respectively Tilburg University (67.2%), Wageningen University (63.3%) and the Free University of Amsterdam (54.5%). Note, however, that only three universities perform significantly differently from the average at the 10% significance level (see Column 5 of Table 5.1): Tilburg University and Wageningen University perform significantly better than (the average of) the other universities, while the opposite conclusion applies for the University of Maastricht.

The bottom panel of Table 5.1 gives the mean efficiency values for the different specialization areas in the sample. Like before, there is considerable efficiency variation over the different specialization areas. In the period 1998-2000, the highest mean efficiency value (of 68.6%) is

areas. However, such a large number of outputs would make the efficiency assessment exercise particularly vulnerable to the so-called 'curse of dimensionality' of nonparametric models, which in this instance would mean a severe upward bias of the efficiency estimates (because we would retain a setting with only 8 university observations for as much as 30 outputs). Therefore, in this study we choose to focus on research programs as production units, and to compute university efficiencies by averaging over the corresponding research programs.

obtained in the field of Spatial and Environmental Economics, which is closely followed by Econometrics (mean efficiency of 66.1%) and Theoretical and Applied Microeconomics (64.8%). The least efficient areas are Applied Labor Economics and Economics of Public Policy (both areas have a mean efficiency value of 35.3%). Five specialization areas performed significantly differently from the mean at the 10% significance level: Econometrics, Theoretical and Applied Microeconomics, and Spatial and Environmental Economics do significantly better than the rest, while research programs in Applied Labor Economics and Economics of Public Policy do significantly worse.

Based on these results, we may conclude that the average performance of universities may largely be driven by different configurations in terms of specialization domains. Indeed, one interpretation of systematic efficiency differences between research programs that are active in different specialization fields is that alternative specializations entail other research production technologies. To correct for this potential bias in our inter-university comparisons, our following exercises focus on systematic differences between universities per specialization type.[17] In addition, accounting for possible flaws in our construction of the input prices/wages, we will analyze these differences by using shadow prices.

Before doing so, we briefly compare our results to those obtained on the basis of a standard cost efficiency measure such as that used by Cherchye and Vanden Abeele (2005; see in particular p. 497-499), which does not incorporate the implications of joint input use and input externalities. For each observation i, this measure is defined as

$$\omega^i = \frac{\min\limits_{j \in D^i} (\mathbf{p}^i)'\mathbf{x}^j}{(\mathbf{p}^i)'\mathbf{x}^i} \text{ with } D^i = \{j \in S \mid \mathbf{y}^j \geq \mathbf{y}^i\}.$$

Referring to our discussion in Section 5.3, we have that $D^i \subseteq \mathbf{R}^i$ and $\omega^i \geq \varphi^i$. Thus, our newly proposed method entails a strengthened efficiency analysis.[18] To interpret this last result in terms of our underlying model of multi-output production, recall from the definition

[17] Cook et al. (1998) provide a general discussion of issues related to DEA efficiency evaluation when the sample can be subdivided into groups. We note that the procedures they present for dealing with grouped samples in the DEA evaluation could also be used in combination with the methodological tools presented in this chapter.

[18] For completeness, we add that Cherchye and Vanden Abeele use shadow prices in their empirical cost efficiency assessment, while the results in Table 5.1 are based on actual price information. We choose to include the cost efficiency results

in Section 5.3 that every output dominating reference set R^i in $\mathbf{R}^i$ contains a combination of observations j^k that each dominate the evaluated observation i in *at least* one output k. The use of the sets reflects that combinations of the input vectors j^k (in R^i), each producing more of the (individual) outputs k than the evaluated input vector i, can also produce the (multi-output) combination $\mathbf{y}^i$. As such, our necessity condition for cost efficient behavior under economies of scope naturally complies with the common intuition that such scope economies imply that the cost of the multi-output production should not exceed the sum of the costs associated with the separate production of the individual outputs. It is essentially this feature, which clearly exploits the multiple output production following from scope economies, that entails the strengthened analysis. For example, the standard cost efficiency measure does not consider combined input vectors: the set D^i (only) contains observations j that dominate observation i in all outputs *simultaneously*.

Table 5.1 reports the results for the measure ω^i in the column 'standard' cost efficiency. Generally, we find that the pattern of the efficiency distribution in that column is similar to that in the column 'scope economies' cost efficiency. For example, we again find that Tilburg University and Wageningen University performed better than the average of the other universities in the period 1998-2000; and we equally obtain that Dutch universities have a comparative advantage in the areas of Spatial and Environmental Economics, Econometrics and Theoretical and Applied Microeconomics. Still, an important observation is that the mean efficiency values obtained by using the newly proposed (scope economies) method are generally lower than those obtained by the standard method. This confirms that an explicit consideration of the features that are specific to multi-output production effectively obtains a more stringent efficiency analysis. In fact, putting an additional *a priori* structure on the decomposed input vectors or implicit price vectors, which includes specific information regarding the nature of the scope economies (in terms of production externalities and/or jointly used inputs), may entail an even stronger analysis.

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based on actual price information, as this enhances the comparison with the results in the column 'scope economies' cost efficiency. Still, the main qualitative conclusions of the shadow price assessment are similar to those obtained on the basis of actual prices.

Table 5.1. Differences between universities and specialization types

	'scope economies' cost efficiency				'standard' cost efficiency	
	numb.	**m.e.**	**st. d.**	**p-value**	**m.e**	**st. d.**
University						
Erasmus University of Rotterdam	59	0.528	0.303	0.831	0.562	0.320
Tilburg University	27	0.672	0.273	0.004	0.741	0.282
University of Nijmegen	6	0.356	0.173	0.156	0.423	0.252
University of Groningen	18	0.437	0.213	0.197	0.537	0.275
University of Maastricht	27	0.417	0.214	0.046	0.469	0.283
University of Amsterdam	35	0.454	0.284	0.136	0.466	0.294
Free University of Amsterdam	36	0.545	0.324	0.597	0.566	0.331
Wageningen University	21	0.633	0.282	0.063	0.622	0.300
Overall	*229*	*0.521*	*0.289*		*0.562*	*0.311*
Specialization area						
Accounting and Finance	36	0.467	0.280	0.224	0.513	0.309
Applied Mathematics	18	0.514	0.284	0.917	0.529	0.299
Development, Growth and	15	0.485	0.279	0.611	0.548	0.317
Transition Econometrics	15	0.661	0.214	0.053	0.733	0.229
Applied Labor Economics	13	0.353	0.155	0.030	0.384	0.205
Marketing and Business Economics	66	0.493	0.291	0.345	0.539	0.315
Macroeconomics, Money and International Issues	18	0.547	0.279	0.693	0.591	0.299
Theoretical and Applied Microeconomics	21	0.648	0.286	0.034	0.692	0.310
Economics of Public Policy	9	0.353	0.219	0.075	0.354	0.218
Spatial and Environmental Economics	18	0.686	0.352	0.011	0.696	0.358
Overall	*229*	*0.521*	*0.289*		*0.562*	*0.311*

Notes: The column p-value reports the (two-sided) probability value for the hypothesis that the mean efficiency of the row categories equals the mean efficiency over all other categories; *m.e.* stands for mean efficiency and *st.d.* for standard deviation.

5.4.2 Differences between universities per specialization type: shadow prices

We next decompose the aggregate performance of each specialization area. More specifically, we evaluate each research program by comparing it to all other research programs *that are active in the same specialization domain* (while -to recall- the results in Table 5.1 follow from comparison to the set of all research programs *independent of their* *specialization type*). Per specialization area, we subsequently calculate the mean efficiency value of each university. This allows us to identify the research domains in which a given university has a comparative advantage as compared to the other universities. This section reports the results of such an exercise when using shadow prices for evaluating the different inputs.[19] That is, for each observation i we concentrate on the cost efficiency measure θ^i introduced in Section 5.3.

We allow different shadow prices for different research programs. In doing so, we effectively account for possible salary differences over universities and specialization areas.[20] Of course, this use of program-specific shadow prices does not account for possible differences in salaries among researchers within one and the same research program; such differences may e.g. follow from different levels of experience, qualifications or productivity. For the current application, it is impossible to account for such differences because we lack the necessary information regarding the composition of the input (= research staff) categories. From that perspective, differences in program-specific shadow prices capture differences in 'average' salaries between research programs (thus reflecting e.g. differences in average experience, qualifications and productivity). At this point, it is also worth recalling the non-uniqueness of the shadow prices computed by means of the linear program in (5.2), whence our following discussion does not focus on the computed shadow prices.

Table 5.2 tabulates the mean efficiency values of Dutch universities per specialization area. The best performing universities per type of specialization are the following: University of Groningen (Accounting and Finance), Tilburg University (Applied Mathematics; Marketing and Business Economics; Theoretical and Applied Microeconomics), Wageningen University (Development, Growth and Transition), Erasmus University (Econometrics and Applied Labor Economics), University of Maastricht (Econometrics, Applied Labor Economics), Free University of Amsterdam (Applied Labor Economics; Spatial and Environmental Economics), University of Amsterdam (Econometrics; Macroe-

[19] We have also conducted similar exercises on the basis of the same (potentially unreliable) price information as before. Generally, this yielded the same qualitative conclusions as for the results in Table 5.2.

[20] Uniform shadow prices across (subgroups of) research programs may be imposed by using the methodological tools that Kuosmanen, Cherchye and Sipiläinen (2006) proposed in a DEA context; these tools are readily adapted to the current set-up.

conomics, Money and International Issues; Economics of Public Policy). In fact, despite the relatively small samples for each specialization domain (ranging from 9 observations to 66 observations), we do find significant efficiency differences for almost all specialization areas (when using the 10% significance level; see the p-values in column 5 of Table 5.2). More specifically, we can identify (at the 10% significance level) universities that do significantly better than the rest (in the fields Marketing and Business Economics; Spatial and Environmental Economics) and, even more importantly, universities that perform systematically worse than other institutes (in the fields Accounting and Finance; Applied Mathematics; Development, Growth and Transition; Marketing and Business Economics; Theoretical and Applied Microeconomics; Spatial and Environmental Economics). This indicates that the presented method may obtain robust conclusions even in the case of small samples, when putting minimalistic a priori structure on the multi-output production process and using 'most favorable' shadow prices for evaluating the different research programs. In this specific application setting, such information may be particularly instrumental for robustly benchmarking the bad performing universities: these institutes may learn from other universities which, within the given specialization area, significantly outperform them.

Next, the above list seems to indicate that most universities have a comparative advantage in at least one specialization area. To some extent, these results are in 'contrast' with the information provided by Table 5.1. Universities that perform well overall (see Table 5.1) may perform relatively badly in some specialization domains, and universities that perform relatively badly overall may perform well in some specialization areas. For example, Tilburg University, which had the highest mean efficiency value in Table 5.1, only performs best in the areas of Applied Mathematics, Marketing and Business Economics and Theoretical and Applied Microeconomics. And Wageningen University, which obtained the second highest overall performance value, only excels in the area of Development, Growth and Transition. This indicates that top universities have some 'core' businesses, in which they reach a generally high performance level. Finally, and not surprisingly, universities that generally perform well do not do (significantly) badly in any of the specialization areas that we consider.

In addition, these results seem to confirm our earlier conjecture that one should take into account technology differences between specialization areas in efficiency analyses, which is in contrast with the more naive (but, apparently, rather widespread) view that one may directly compare the performance of research programs that are active in very different specialization areas within the general Economics profession. In fact, disaggregating over specialization domains seems a necessity when assessing the research efficiency of universities: examining aggregate faculty figures, which is conventional practice, does not always provide useful insights in terms of the aim of increasing a university's performance. Quite the contrary: disaggregated figures allow us to situate the comparative advantage of different institutes, which, in turn, can lead to further performance improvements through specialization-specific research policies. As such, this conclusion provides further support for Cherchye and Vanden Abeele's (2005) motivation to focus on micro-units of research production, such as research programs, rather than on macro-units, such as university faculties. From that perspective, the results in Table 5.2 provide useful complementary information to the results of Cherchye and Vanden Abeele (discussed in the previous section), by specifically considering efficiency differences between universities within one and the same specialization domain. As we indicated above, such results may be useful, for example, from a benchmarking perspective.

Table 5.2. Differences between universities per specialization type; efficiency differences based on shadow prices

	numb.	mean eff.	st. dev.	p-value
Accounting and Finance				
Erasmus University of Rotterdam	6	0.826	0.357	0.481
Tilburg University	6	0.831	0.171	0.457
University of Groningen	3	1.000	0.000	0.146
University of Maastricht	6	0.722	0.309	0.886
University of Amsterdam	9	0.576	0.365	0.078
Free University of Amsterdam	6	0.696	0.385	0.720
Overall	*36*	*0.740*	*0.322*	
Applied Mathematics				
Erasmus University of Rotterdam	9	0.787	0.300	0.080
Tilburg University	3	1.000	0.000	0.344
University of Amsterdam	3	0.970	0.052	0.482
Free University of Amsterdam	3	0.959	0.071	0.541
Overall	*18*	*0.881*	*0.230*	
Development, Growth and Transition				
University of Groningen	3	0.999	0.001	0.293
University of Maastricht	3	0.348	0.070	0.000
Free University of Amsterdam	6	0.950	0.088	0.249
Wageningen University	3	1.000	0.000	0.291
Overall	*15*	*0.849*	*0.267*	
Econometrics				
Erasmus University of Rotterdam	3	1.000	0.000	0.460
Tilburg University	3	0.958	0.073	0.117
University of Maastricht	3	1.000	0.000	0.460
University of Amsterdam	3	1.000	0.000	0.460
Free University of Amsterdam	3	0.973	0.028	0.481
Overall	*15*	*0.986*	*0.035*	
Applied Labor Economics				
Erasmus University of Rotterdam	2	1.000	0.000	0.689
University of Maastricht	3	1.000	0.000	0.606
University of Amsterdam	5	0.982	0.040	0.220
Free University of Amsterdam	3	1.000	0.000	0.606
Overall	*13*	*0.993*	*0.025*	
Macroeconomics, Money and International Issues				
Erasmus University of Rotterdam	6	0.942	0.102	0.188
Tilburg University	3	0.998	0.004	0.442
University of Nijmegen	3	0.995	0.009	0.498
University of Maastricht	3	0.948	0.051	0.521
University of Amsterdam	3	1.000	0.000	0.405
Overall	*18*	*0.971*	*0.064*	

	numb.	**mean eff.**	**st. dev.**	**p-value**
Marketing and Business Economics				
Erasmus University of Rotterdam	18	0.766	0.238	0.483
Tilburg University	9	0.905	0.158	0.033
University of Nijmegen	3	0.544	0.228	0.228
University of Groningen	12	0.589	0.271	0.047
University of Maastricht	9	0.713	0.268	0.857
University of Amsterdam	3	0.474	0.124	0.095
Free University of Amsterdam	6	0.715	0.390	0.904
Wageningen University	6	0.881	0.243	0.145
Overall	*66*	*0.728*	*0.269*	
Theoretical and Applied Microeconomics				
Erasmus University of Rotterdam	6	0.808	0.298	0.612
Tilburg University	3	1.000	0.000	0.199
University of Amsterdam	3	0.607	0.198	0.036
Free University of Amsterdam	3	0.966	0.059	0.324
Wageningen University	6	0.872	0.159	0.755
Overall	*21*	*0.848*	*0.219*	
Economics of Public Policy				
Erasmus University of Rotterdam	6	0.948	0.124	0.506
University of Amsterdam	3	1.000	0.000	0.506
Overall	*9*	*0.965*	*0.102*	
Spatial and Environmental Economics				
Erasmus University of Rotterdam	3	0.401	0.213	0.000
University of Amsterdam	3	0.928	0.125	0.609
Free University of Amsterdam	6	1.000	0.000	0.087
Wageningen University	6	0.907	0.185	0.566
Overall	*18*	*0.857*	*0.251*	

Notes: The column p-value reports, for each specialization type, the (two-sided) probability value for the hypothesis that the mean efficiency of the row categories equals the mean efficiency over all other categories.

5.5 Summary and concluding remarks

We have presented a nonparametric methodology for analyzing the cost efficiency of firms that produce multiple outputs. Our starting point is that such multi-output production basically refers to economies of scope in the production process, which in turn refers to joint input use and input externalities. Given this, we have instituted a nonparametric characterization of efficient behavior under these general conditions, and subsequently derived necessary and sufficient empirical conditions for data consistency with the cost efficiency requirement. Importantly, these conditions only include observed firm demand and supply data; this means that inputs are not to be disaggregated in terms of the specific channels through which they can be allocated (i.e., to a specific output or to joint use for the production of different outputs). Essentially, we have designed cost efficiency conditions that exploit the implications of scope economies (through joint input use and input externalities) at the level of the observable aggregate prices and quantities. In addition, we have relaxed the assumption that input prices are observed, to come up with (linear programming) efficiency tests that utilize (implicit) shadow prices. *Inter alia*, this provides a direct link with the nonparametric efficiency assessment literature known as Data Envelopment Analysis (DEA).

We have illustrated our methodology by examining the cost efficient behavior of research programs in Economics and Business Management faculties of Dutch universities. This application shows that the proposed methodology is easy to implement in practice, even for fairly large data sets (e.g., our application involved 229 observations). In fact, we recall from our discussion in Section 5.4 that exploiting scope economies entails a strengthened cost efficiency analysis as compared to more conventionally used alternatives, such as in Cherchye and Vanden Abeele (2005). As we explain below, the efficiency evaluation can be strengthened even further by putting additional structure on the decomposed input vectors and implicit price vectors.

In addition, our application demonstrates the practical usefulness of the method for obtaining robust conclusions regarding cost efficiency differences between universities within specific specialization areas, also when using shadow prices to evaluate the different inputs. As we have indicated, in our specific application set-up such insights may be particularly useful for benchmarking purposes.

A general qualitative conclusion of our results is that they seem to support the necessity of accounting for technological differences between specialization domains (within the general Economics profession) when analyzing research performance. Given this, we have analyzed performance differences between universities at the level of individual specialization areas. We found that universities indeed seem to specialize in only a few research domains: while universities that perform best overall generally perform well in all the domains in which they are active, universities that are generally less efficient can also perform very well in certain areas of specialization. The fact that many of these findings turned out to be statistically significant illustrates that our method can obtain robust conclusions while imposing a minimalistic *a priori* structure on the actual (but unknown) production process (even in the cases where we could only use small (specialization-specific) data sets).

At this point, it is worth stressing the limitations of our empirical analysis, which mainly served to illustrate the proposed methodology. Most importantly, we did not explicitly account for errors-in-variables and small sample bias. Still, we want to indicate that (i) we have used an input-output configuration that largely coincides with that considered by the VSNU in their quinquennial assessment based on the same data, and (ii) the data, which were reported by the universities themselves in extensive self-assessments, are relatively well standardized and have been subjected to some scrutiny for correctness and consistency, which gives us reasonable confidence in their quality. But in order to draw absolute conclusions from our exercises, it seems recommendable to utilize methodological tools that satisfactorily deal with errors-in-variables. See, for example, Grosskopf (1996) for a survey of tools that are currently available in the nonparametric literature, and Cazals, Florens and Simar (2002) for a more recent proposal; these tools (that were originally proposed in a DEA context) may easily be accommodated to the newly proposed cost efficiency assessment methodology. Next, the sampling problem may apply in particular to our specialization-specific exercises, which are often based on a fairly limited number of observations. To obtain more robust results in such cases, one may for example use the bootstrap procedure proposed (again, in a DEA context) by Simar and Wilson (1998), which is also readily adapted to the presented efficiency evaluation tools.

Three concluding remarks are in order. First, to keep the discussion focussed, we have concentrated on consistency testing and the associated efficiency measurement. Still, Varian (1984) emphasized alternative uses of the nonparametric approach in addition to testing for optimizing firm behavior, namely recovering the production set and forecasting firm behavior under new price conditions. It is worth emphasizing that such recoverability and forecasting questions may also be addressed when starting from the specific (scope economies-based) condition for cost efficiency that has been forwarded in this study; the methodological extensions develop along directly analogous lines as in

Varian (1984). The goodness-of-fit and subset rationalization concepts of, respectively, Varian (1990) and Banker and Maindiratta (1988) allow for addressing such questions while accounting for observed inefficiencies.

Another remark pertains to the shadow price efficiency measurement problem in (5.2), which is applicable when reliable price information is not available. Such a shadow price analysis can be strengthened by imposing price information in the form of additional constraints that define a feasible range for the relative prices, which may rule out the extreme cases where the relative price of a commodity approaches zero or infinity. The technical questions related to incorporating such price restrictions have been discussed extensively in a DEA context, most commonly under the label 'weight restrictions' or 'assurance regions' (see, e.g., Allen et al., 1997; Pedraja-Chaparro et al., 1997, for surveys; and Kuosmanen et Cherchye and Sipiläinen, 2006, for more recent developments). These tools may be adapted to the current set-up.

Finally, from a related perspective, putting an additional *a priori* structure on the implicit input prices and quantities will obviously strengthen the cost efficiency analysis of the multi-output production process at hand. Extra structure on the decomposed quantities may, for example, reflect additional information (or assumptions) regarding the presence of jointly used inputs, or regarding (shares of) inputs that are specifically used for the production of particular outputs; similarly, additional structure on the implicit prices may reflect some *a priori* position regarding the presence of externalities in the production of certain outputs. Generally, such extra price-quantity conditions will entail refinements of the general model presented in Section 5.2 which in turn will lead to more stringent necessary and sufficient cost efficiency conditions in terms of observables (see Section 5.3). These conditions may be obtained along similar lines as in the proofs of Propositions 5.6 and 5.8.

In this respect, it is also interesting to compare our approach with Activity Based Costing (ABC) (e.g., Cooper and Kaplan, 1988; Christensen and Demski, 1995, elaborate on the relationship between ABC and the classical theory of cost). Essentially, ABC uses information regarding the input costs (= prices * quantities) that are (through so-called 'cost drivers') allocated to individual outputs. By contrast, our approach does not require such information, but starts from a separate allocation of input quantities (resulting in decomposed input vectors) and input prices (resulting in implicit price vectors) to different outputs. From this perspective, ABC can be considered complementary to our approach: the input cost information used in ABC can be useful for putting a priori restrictions on the feasible combinations of decomposed input vectors and implicit price vectors.

In summary, the approach to modeling multi-output production presented here provides a general framework for a wide spectrum of production models that incorporate input externalities and joint input use.

Appendix

Proof of Proposition 5.4

(i; necessity) Each bundle $(\widehat{\mathbf{x}}^i)' = (\,(\mathbf{x}_1^i)' \cdots (\mathbf{x}_s^i)' \,(\mathbf{x}_{s+1}^i)'\,)'$ solves the following problem for given priority weight vector μ^i :[21]

$$\max_{\mathbf{x}_1^i,\ldots,\mathbf{x}_{s+1}^i} \sum_{k\in K} \mu_k^i f_k(\mathbf{x}_1^i, \ldots, \mathbf{x}_{s+1}^i) \text{ s.t.} (\mathbf{p}^i)'(\mathbf{x}_1^i + \cdots + \mathbf{x}_{s+1}^i) \leq (\mathbf{p}^i)'\mathbf{x}^i.$$

Given concavity, the output-specific production functions are subdifferentiable, which carries over to their weighted sum $\sum_{k\in K} \mu_k^i\ f_k$. An optimal solution to the above maximization problem should therefore satisfy (for $l = 1, \ldots, s+1$)

$$\sum_{k\in K} \mu_k^i f_k^{\mathbf{x}^l} \leq \eta^i \mathbf{p}^i,$$

for η^i the Lagrange multiplier associated with the cost constraint, and $f_k^{\mathbf{x}^l}$ a subgradient of the production function f_k defined for the vector $\mathbf{x}^l$ and evaluated at $(\,(\mathbf{x}_1^i)' \cdots\ (\mathbf{x}_s^i)' \,(\mathbf{x}_{s+1}^i)'\,)'$. Letting $\mathfrak{p}_{k,l}^i = \frac{\mu_k^i f_k^{\mathbf{x}^l}}{\eta^i}$ and $\lambda_k^i = \frac{\eta^i}{\mu_k^i}$ thus gives (for $l = 1, \ldots, s+1$)

$$f_k^{\mathbf{x}^l} \leq \lambda_k^i \mathfrak{p}_{k,l}^i. \tag{5.3}$$

Next, concavity of the functions f_k implies for all $k \in K$

$$f_k(\mathbf{x}_1^j, \ldots, \mathbf{x}_{s+1}^j) - f_k(\mathbf{x}_1^i, \ldots, \mathbf{x}_{s+1}^i) \leq \sum_{l=1}^{s+1} (f_k^{\mathbf{x}^l})'(\mathbf{x}_j^l - \mathbf{x}_i^l). \tag{5.4}$$

Substituting (5.3) in (5.4) and setting $y_k^i = f_k(\mathbf{x}_1^i, \ldots, \mathbf{x}_{s+1}^i)$ obtains

$$y_k^j - y_k^i \leq \lambda_k^i \sum_{l=1}^{s+1} (\mathfrak{p}_{k,l}^i)'(\mathbf{x}_l^j - \mathbf{x}_l^i), \tag{5.5}$$

[21] To be exact, we have that f_k is quasi-concave rather than concave. For compactness, however, we consider f_k concave in the following proof. Indeed, under mild regularity conditions and strict convexity of the upper-level sets (which follows from $\mathbf{p}^i \in \mathbb{R}_{++}^m$ in our case), for any quasi-concave function there exists a positive monotone transformation that obtains a concave representative. Such a monotone transformation is harmless in view of the equivalence between (5.6) and (5.5), which applies for any monotonous transformation of f_k.

which is equivalent to

$$y_k^j \geq y_k^i \Rightarrow \sum_{l=1}^{s+1} (\mathfrak{p}_{k,l}^i)' \mathbf{x}_l^j \geq \sum_{l=1}^{s+1} (\mathfrak{p}_{k,l}^i)' \mathbf{x}_l^i. \tag{5.6}$$

(ii; sufficiency) Recalling the equivalence between (5.6) and (5.5), we first define for $(k = 1, \dots s)$

$$f_k(\mathbf{x}_1, \dots, \mathbf{x}_{s+1}) = \min_{i \in S} [y_k^i + \lambda_k^i \sum_{l=1}^{s+1} (\mathfrak{p}_{k,l}^i)' (\mathbf{x}_l - \mathbf{x}_l^i)] \tag{5.7}$$

Varian (1984, Theorem 2) proves that $f_k(\mathbf{x}_1, \dots, \mathbf{x}_{s+1}) = y_k^i$. Next, given $\mu^i \in \mathbb{R}_+^s$, we have for all $\widehat{\mathbf{x}}^i$ such that $(\mathbf{p}^i)'(\mathbf{x}_1 + \dots + \mathbf{x}_{s+1}) \leq (\mathbf{p}^i)' \mathbf{x}^i$

$$\sum_{k \in K} \mu_k^i \ f_k(\mathbf{x}_1, \dots, \mathbf{x}_{s+1}) \leq \sum_{k \in K} \mu_k^i \ [y_k^i + \lambda_k^i \sum_{l=1}^{s+1} (\mathfrak{p}_{k,l}^i)' (\mathbf{x}_l - \mathbf{x}_l^i)].$$

Without losing generality, we concentrate on $\mu_k^i = (\lambda_1^i / \lambda_k^i)$, which obtains

$$\sum_{k \in K} \mu_k^i \ f_k(\mathbf{x}_1, \dots, \mathbf{x}_{s+1}) \leq \sum_{k \in K} \mu_k^i y_k^i + \lambda_1^i [(\mathbf{p}^i)' (\sum_{l=1}^{s+1} (\mathbf{x}_l - \mathbf{x}_l^i))].$$

Since $(\mathbf{p}^i)'(\mathbf{x}_1 + \dots + \mathbf{x}_{s+1}) \leq (\mathbf{p}^i)' \mathbf{x}^i$, we thus have

$$\sum_{k \in K} \mu_k^i \ f_k(\mathbf{x}_1, \dots, \mathbf{x}_{s+1}) \leq \sum_{k \in K} \mu_k^i \ y_k^i = \sum_{k \in K} \mu_k^i \ f_k(\mathbf{x}_1^i, \dots, \mathbf{x}_{s+1}^i),$$

which proves that $\widehat{\mathbf{x}}^i$ maximizes $\sum_{k \in K} \mu_k^i \ f_k(\mathbf{x}_1, \dots, \mathbf{x}_{s+1})$ subject to $(\mathbf{p}^i)'(\mathbf{x}_1 + \dots + \mathbf{x}_{s+1}) \leq (\mathbf{p}^i)' \mathbf{x}^i$. We conclude that the functions $f_k(\mathbf{x}_1, \dots, \mathbf{x}_{s+1}) (k \in K)$ in (5.7) provide a *C-R* of the data. These functions are concave, monotonously increasing and continuous (see Varian, 1984, Theorem 2). The corresponding input requirement sets are closed, convex and positive monotonous. *Q.E.D.*

Proof of Proposition 5.6

For any set R^i with $\forall k \in K, \exists j^k \in R^i : y_k^{j^k} \geq y_k^i$ consistency with the C-R conditions in Proposition 5.4 requires for all k

$$y_k^{j^k} \geq y_k^i \Rightarrow \sum_{l=1}^{s+1} (\mathfrak{p}_{k,l}^i)' \mathbf{x}_l^{j^k} \geq \sum_{l=1}^{s+1} (\mathfrak{p}_{k,l}^i)' \mathbf{x}_l^i$$

and thus

$$\sum_{k \in K} (\sum_{l=1}^{s+1} (\mathfrak{p}_{k,l}^i)' \mathbf{x}_l^{j^k}) \geq \sum_{k \in K} (\sum_{l=1}^{s+1} (\mathfrak{p}_{k,l}^i)' \mathbf{x}_l^i).$$

The result then follows from the fact that

$$\sum_{j^k \in R^i} (\mathbf{p}^i)' \mathbf{x}^{j^k} \geq \sum_{k \in K} (\sum_{l=1}^{s+1} (\mathfrak{p}_{k,l}^i)' \mathbf{x}_l^{j^k})$$

and $\sum_{k \in K} (\sum_{l=1}^{s+1} (\mathfrak{p}_{k,l}^i)' \mathbf{x}_l^i) = (\mathbf{p}^i)' \mathbf{x}^i$. *Q.E.D.*

Proof of Proposition 5.8

Suppose that a construction $S_k, k \in K$, consistent with the sufficiency condition in Proposition 5.8 exists. Given this, we can construct a configuration of unobservable, decomposed input vectors and implicit price vectors that meet the C-R conditions in Proposition 5.4. Specifically, we use

for $i \in S: \ \mathfrak{p}_{k,k^*}^i = \mathbf{p}^i$ for $k = k^*$, $\mathfrak{p}_{k,k^*}^i = \mathbf{0}$ for $k \neq k^*$; and
for $i \in S_k : \mathbf{x}_k^i = \mathbf{x}^i$.

We obtain the sufficiency result in two steps. First, for $i \in S_k$ we have for any $j \in S$: if $y_k^j \geq y_k^i$ then $j \in S_k$. (Indeed, $j \in S_{k^*}, k^* \neq k$ would require $y_k^j < y_k^i$, which is not the case.) Given this, we can distinguish two cases for each $i, j \in S$ with $y_k^j \geq y_k^i$. If $i \in S_k$ then $j \in S_k$ (see before) and consequently $(\mathbf{p}^i)' \mathbf{x}^i = \sum_{l=1}^{s+1} (\mathfrak{p}_{k,l}^i)' \mathbf{x}_l^i \leq \sum_{l=1}^{s+1} \mathfrak{p}_{k,l}^i \mathbf{x}_l^j = (\mathbf{p}^i)' \mathbf{x}^j$ by construction. Alternatively, if $i \notin S_k$ we have $\sum_{l=1}^{s+1} (\mathfrak{p}_{k,l}^i)' \mathbf{x}_l^i = 0$ and the condition $0 = \sum_{l=1}^{s+1} \mathfrak{p}_{k,l}^i \mathbf{x}_l^i \leq \sum_{l=1}^{s+1} \mathfrak{p}_{k,l}^i \mathbf{x}_l^i$ is always satisfied. *Q.E.D.*

General conclusion

The main objective of this dissertation was to develop and apply nonparametric tests of general collective choice behavior for both consumption and production settings.

In Chapter 2, we generalized the work of Chiappori (1988, 1992) by providing a nonparametric characterization of the general consumption model of Browning and Chiappori (1998), which includes public consumption and (positive) externalities. Starting from (only) aggregate household data, this characterization led to testable collective rationality conditions which have a structure similar to the GARP conditions, which, to recall, form the basis for nonparametric tests of individual rationality.

Our general model encompasses a large variety of alternative behavioral models as special cases; these cases mostly boil down to restrictions on the feasible personalized quantities and prices. Besides considering tests of the general collective model, we also investigated such special models in our empirical applications in Chapters 3 and 4.

In our real-life application in Chapter 3 we started by describing an efficient algorithm for our necessity condition (Proposition 2.7). We used this algorithm for analyzing the (necessity constraints for) collective rationality of couples that were drawn from the Russia Longitudinal Monitoring Survey. In this way we obtained a first empirical application of nonparametric tests of the general collective model.

Since all couples passed our necessity test, we then developed a nonparametric framework for restricting the collective model by constraints on the sharing rule. To recall, the sharing rule defines the within household distribution of the household means. In our application we obtained that a multitude of collective consumption models were able to describe the collective behavior. On the other hand, the unitary model, which can be seen as limiting case of our restricted collective models, was not able to rationalize the observed behavior.

We ended the chapter by introducing two power measurements from which we concluded that the (general) collective rationality tests are (rather) powerful at the sample level but have less bite on the household level. While the first power is useful in tests that compare the adequacy of several behavioral models (e.g. unitary versus collective model), the second power reflects how good we will be able to tackle recovery and forecasting issues. Indeed, a low power at the household level, will generally imply imprecise recovery and forecasting results. However, as we will discuss below, there are several avenues to significantly increase the power at the household level.

A first example is given in Chapter 4 where we considered another class of restricted collective models by focusing on the 'egoistic model' (i.e. dyads consisting of egoistic individuals and excluding public consumption). As we showed in our empirical application, the parsimonious nature of this model allows for a more powerful analysis. The test results concerning the choice behavior of dyads in our simple consumption setting, provided strong support for this restricted model. For those dyads that did not pass the test, we showed consistency with our general model; as such we provided an interpretation in terms of externalities and/or public consumption.

An important feature of this second application is that it uses laboratory data which allows us to avoid usual data problems. Moreover this setting enables us to obtain information on the individual consumption quantities. Again this resulted in an increased power of our tests.

Our empirical application in Chapter 4 is a first example of a nonparametric test of collective rationality on experimental data. As such our study complements the existing nonparametric-experimental literature that focuses on individual rationality (see, e.g., Sippel, 1997, Harbaugh, Krause and Berry, 2001, Andreoni and Miller 2002). We believe that nonparametric consumption analysis of experimental data can be very useful for gaining insight in the mechanics of the intrahousehold decision process (or more generally, in the mechanics of group decision making).

In the Chapters 2 to 4 we focused on consumption behavior, while in the final Chapter 5 we analyzed choice behavior in a production setting. More precisely, we presented a nonparametric methodology for analyzing the cost efficiency of firms that benefit from economies of scope. Our starting point is that such multi-output production refers to joint input use and input externalities. Given this, we exploited the conceptual analogy with the collective consumption model in Chapter 2 to develop a nonparametric characterization of cost minimizing behavior of multi-output firms. This resulted in operational necessity and sufficiency tests that are solely based on observed aggregate demand and supply data. In addition, we also relaxed the assumption of observed input prices and introduced shadow prices in our efficiency tests. As such we directly linked our tests to the nonparametric efficiency assessment literature known as Data Envelopment Analysis. Finally, we also used our tests to analyze the cost efficient behavior of research programs in Economics and Business Management faculties of Dutch universities. In our empirical application we obtained robust results and showed that our approach entailed a strengthened cost efficiency analysis.

The above summary of our dissertation makes clear that we focused on developing and applying nonparametric tests of collective choice behavior. These results will allow us to address welfare related questions concerning the intrahousehold (or more general intragroup) allocation process (see Chapter 1 for motivating example questions). However, as argued before, the characterization of the observed household behavior is only a first step in answering these questions. In our future work, we can now focus on the second complementary step: recovery of the decision structure underlying the observed household behavior (e.g. the individual preferences and the sharing rule) and forecasting the household behavior in new situations (see, e.g., Cherchye, De Rock and Vermeulen, 2007d). To obtain these results, we can exploit the similarity of our tests with the GARP conditions, in order to adapt existing result for individual rationality towards the collective model (see, e.g., Varian, 1982, 1983 and 2006, and Blundell, Browning and Crawford, 2003 and 2007, for inspiring results). In Chapter 3 we already did this for power measurements of Bronars (1987) and in Chapter 4 for Varian's goodness-of-fit measure (see Varian, 1990).

From the empirical applications in Chapters 3 and 4, we may conclude that the power of our tests of the general collective model is rather low. However, there are several possibilities to increase the power of our results. Firstly, as we already did in our empirical applications, we can further consider restricted collective models in order to obtain more

stringent tests. Such restrictions could for example be: observability of the distribution of goods (or household means over) the household members (e.g exclusive goods/assignable information and sharing rule restrictions); or imposing structure on the nature of the externalities (e.g. exclude externalities for some (all) goods). Our results, and the current literature, suggest that this is a fruitful strategy.
Secondly, our empirical applications were illustrative and considered rather small data sets. Obviously, if we want to increase the power of our results, we have to increase the sample size. In Cherchye, De Rock, Sabbe and Vermeulen (2007), we therefore develop an alternative algorithm, based on integer programming, which allows us to deal with larger data sets.
Finally, we may try to adapt the 'sequential maximum power path' idea from Blundell, Browning and Crawford (2003, 2007), which is formulated in an unitary setting, towards our collective model. Using this concept, these authors obtain powerful recovery results for their unitary setting.

The above remarks pertain to consumption settings, but of course, analogous remarks hold for production settings. Firstly, we can further apply our insights of the consumption context to the production setting, in order to extend the existing nonparametric toolkit for analyzing cost efficient behavior (see, e.g., Varian, 1984, and Banker and Maindiratta, 1988 for inspiring examples). We could for instance consider incorporating information on the decomposed input vectors and/or implicit price vectors to obtain more powerful results. Secondly, as argued before, our tests extend the current Data Envelopment Analysis (DEA) literature. We may therefore try to incorporate known results for DEA, on for instance including restrictions on the implicit prices, in our tests. See, e.g., Allen et al. (1997) and Pedraja-Chaparro et al. (1997) for surveys; and Kuosmanen, Cherchye and Sipiläinen (2006) for more recent developments.
Finally, to increase power we can also apply the nonparametric bootstrap methodology suggested by Simar and Wilson (1998) in a nonparametric production context.

Once we have such a more powerful nonparametric machinery, we may try to gain insight in the mechanics of collective choice behavior under alternative restrictions. For the analysis of the household (or more general group) behavior, we can do this, like in Chapter 4, by using laboratory data to test different assumptions concerning for instance the preferences and/or the decision process. Given that such a setting optimally controls for heterogeneity and data problems, we may expect to obtain more robust results. Moreover, the experimental set-up allows us to obtain extra information concerning for instance the sharing rule or the nature of the externalities.

Besides investigating alternative restrictions for the current (static) setting, this machinery also allows us to tackle other intriguing issues suggested in this work. In this respect, we will firstly focus on integrating measurement error into our nonparametric tests. Cherchye, De Rock, Sabbe and Vermeulen (2007) discuss the possibility to use their IP formulation to include the idea of Varian (1985) for dealing with measurement error. For applications of this idea in an unitary setting, see, e.g., Blundell, Browning and Crawford (2007) and Crawford (2007). Secondly, we may also try to integrate intertemporal aspects in our nonparametric tests such as changing preferences (e.g. habit formation) and intertemporal decision making. See, e.g., Browning (1989), Crawford (2007) for existing nonparametric results in an unitary setting and Mazzocco (2007) for parametric results in a collective setting.

Finally, let us return once more to the policy relevant questions. As already emphasized before, many of these questions are related to the bargaining weights (e.g. measuring the effect of targeting a benefit to specific members in the household). So within the collective model, recovery concerns not only the underlying preferences of individual household members, but mostly it aims to retrieve the underlying ‘weights’, which are accorded within the decision process. Similarly, forecasting pertains not only to the consumption quantities but also to the individuals’ weights that apply in newly defined situations. So, a final line of future research aims to further develop the methodology for retrieving these weights and to tackle as such policy relevant questions as the ones introduced in Chapter 1 (see again Cherchye, De Rock and Vermeulen, 2007d).

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